The cosmic neutrinos discovered by IceCube are doubly unique: they have the highest detected neutrino energies --- up to a few PeV --- and travel the longest distances --- up to a few Gpc, the size of the observable Universe. These features make them attractive probes of particle-physics properties, possibly tiny in size, at energy scales unreachable by other means. In the decades before the IceCube discovery there were plenty of proposals of prospective studies using high-energy cosmic neutrinos. Today, these proposals have become a reality. I will showcase examples of testing neutrino physics at these scales, including stringent tests of physics beyond the Standard Model, like new neutrino-neutrino interactions and neutrino decay.