Vortices in Superconductors

Wilson A. Ortiz
Physics Department
Univ. Federal de São Carlos

wortiz@df.ufscar.br
Let's get started with this Tutorial on Vortices in Superconductors
1. Superconductivity
2. Vortices in SCs
3. Magneto-optical imaging (MOI)
Superconductivity, a macroscopic quantum phenomenon discovered more than one century ago, is a field including a huge variety of materials, many of which have encountered relevant practical applications.
The first part of this Tutorial will be devoted to briefly review the history of Superconductivity, followed by an introductory discussion of the main features of superconducting materials and their uses in real life.
In the second part we’ll discuss vortices in superconductors: occurrence, dynamics, implications for applications.

Before finishing we’ll devote some time to the MOI technique employed in our lab.
I. Superconductivity
A bit of history

1913 Nobel Prize in Physics

H. Kamerlingh Onnes (Leiden, NL)

1908 - Liquid Helium

1911 - Superconductivity in Hg, $T_c \sim 4.2$ K

1913 - Superconductivity in Pb, $T_c \sim 7.2$ K

Figure 2.14 Resistivity-versus-temperature plot obtained by Kamerlingh Onnes when he discovered superconductivity in Leiden in 1911.
A bit of history...

1933 - Meissner effect: perfect diamagnetism

\[ B = \mu_0 (H + M) = 0 \Rightarrow M = -H \]
A bit of history...

1950's - Fritz e Heinz London ($\lambda$); Pippard ($\xi$)

1950 - Phenomenological Theory proposed by Ginzburg and Landau

1957 - Microscopic Theory by Bardeen, Cooper and Schrieffer (BCS)
   Cooper pairs (bosons) - Phys.Rev.104 (1956)
   Boson condensate - Phys.Rev.108 (1957); Nobel Prize (1972)

1959 - Gor'kov: GL can be derived from BCS
1957 - Abrikosov predicted the existence of another type of SC (type II)

Surface energy can be negative in certain cases \((\kappa = \frac{\lambda}{\xi} > 0.707)\)

\(\Rightarrow\) creation of N/SC interfaces becomes energetically favorable

Fluxoides or Vortices: normal regions in the form of "tubes", penetrated by one quantum flux each, \(\phi_o \sim 2 \times 10^{-15} \text{ SI},\) surrounded by superconducting screening currents.
**Vortex matter**

\[ \int B \, dA = \frac{h}{2e} = \Phi_0 \]

**Flux quantum**

- **Type-I**
- **Type-II**

**Vortices give guidance**

Landau’s pupil, Alexei Abrikosov, realised almost immediately that Ginzburg and Landau’s theory can also describe those superconductors (type II) that can coexist with strong magnetic fields. According to Abrikosov’s theory this occurs because the superconductor allows the magnetic field to enter through vortices in the electron superfluid. These vortices can form regular structures, Abrikosov lattices, but disordered structures can also occur.

**Alexei A. Abrikosov**
Argonne National Laboratory, Argonne, Illinois, USA

An Abrikosov lattice of vortices in a type-II superconductor. The magnetic field passes through the vortices.
A bit of history...

Primeira Imagem
Bitter Decoration 1967
Pb-4at%In rod, 1.1K, 195G
U. Essmann and H. Trauble
Max-Planck Institute, Stuttgart

Bitter Decoration
YBa$_2$Cu$_3$O$_7$ crystal, 4.2K, 52G
P. L. Gammel et al., Bell Labs

Scanning Tunnel Microscopy
NbSe$_2$, 1T, 1.8K
H. F. Hess et al., Bell Labs

Scanning Hall probes
YBaCuO film, 1000G
A. Oral et al.
University of Bath

Magneto-Optical Imaging
NbSe$_2$ crystal, 4.3K, 3G
P.E. Goa et al.
University of Oslo
A bit of history...

1986 - Age of the “High Temperature Superconductors” - HTS (type II)

17/04/86 - Bednorz & Muller - \( \text{Ba}_x\text{La}_{5-x}\text{Cu}_5\text{O}_y \) - \( T_c \sim 30 - 35 \text{ K} \)

1987 - Chu, Zhao - \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \) (\( \text{YBCO}, \text{YBaCuO}, \text{Y-123} \)) - \( T_c \sim 92 \text{ K} \)

1988 - \( \text{Bi}_2\text{Ba}_2\text{CaCu}_2\text{O}_y \) - \( T_c \sim 110 \text{ K} \)

1993 - \( \text{Tl}_2\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10-x} \) - \( T_c \sim 132 \text{ K}, \) \( \text{HgBa}_2\text{Ca}_n\text{Cu}_{n+1}\text{O}_{2n+4} \) - \( T_c > 130 \text{ K}, \)

Great hopes for applications partially frustrated: ceramic materials are difficult to mold and, moreover, critical currents are limited by weak-links.
Figure 24 The observed superconducting transition temperature ($T_c$) of a variety of classes of superconductors is plotted as a function of time. Recent discoveries have increased the highest-observed $T_c$ in a number of materials to unprecedented levels, such as in heavy fermion (PuCoGa$_5$), carbon nanotubes (CNTs), and graphite intercalated compounds (CaC$_6$).
Wide list of applications:

Energy - production, storage & distribution;

Sensing magnetic fields;

Production of strong magnetic fields for:
- Nuclear Magnetic Resonance (research)
- NMR Imaging (medical use)
- Deflection, focusing and detection of charged particle beams (particle accelerators)
- Plasma confinement (fusion reactors)
- Levitation (transport of load and people)
Some applications

Current Limiters (transmission lines)

Qubits (quantum computing)

Wires for solenoids – generation of high magnetic fields (labs; NMRs)
Some applications

SQUIDs - flux detectors (labs; magnetographies: encephalo-, cardio-)
Some applications

Magnetic levitation:
- Bearings for large rotors (fly wheels)
- Maglev prototype (Japanese, Nb-Ti)
- Test vehicle (Chinese, HTS)
- HTS SC cables - Second Generation

Voltage regulating systems of Eolic Generators
Superconductivity is a Macroscopic Quantum State featuring two distinguishing properties:

. Supercurrents (dissipationless transport)

. Screening of magnetic fields (Meissner effect)
Zero-voltage supercurrents

Two classics: H. K. Onnes and Bednorz & Müller

H. Kamerlingh Onnes

Hg - 1911

Bednorz & Müller

La$_{2-x}$Ba$_x$CuO$_4$ - 1986
At any given temperature (T) and applied magnetic field (H), a superconducting sample is able to carry a maximum supercurrent density, $J_c$, the Critical Current:

$$J_c = J_c (H, T)$$

**Figure 6:** Four-terminal current-voltage characteristics of the Al film $V(I)$ at various magnetic fields $H$.

*Scientific Reports 3, Article number: 2274 doi:10.1038/srep02274*
At any given temperature (T) and applied magnetic field (H), a superconducting sample is able to carry a maximum supercurrent density, $J_c$, the Critical Current:

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Superconductivity - Basic Concepts

Superconductivity is a Macroscopic Quantum State featuring two distinguishing properties:

- Supercurrents (dissipationless transport)
- Screening of magnetic fields (Meissner effect)
Screening of magnetic fields - Meissner effect

Zero field cooling (ZFC)  Field-cooling (FC)

http://www.fys.uio.no/super/
Magnetic Field Screening

M \quad M

\begin{align*}
J_c & \propto \Delta M \\
H_c & \quad H_c \\
H_{ir} & \\
H & \quad H
\end{align*}

(type II superconductor)
Penetration Profile: Critical State

type II supercondutor

\[ M \]

\[ H \]

\[ J_c \propto \Delta M \]

\[ H_{c1} \]

\[ H_{c2} \]

\[ H_{ir} \]
Flux distribution apparently continuous... in reality, quantized flux: vortices

http://www.fys.uio.no/super/
Although quantized, flux is usually (conveniently) treated as “continuous”

Bean Model

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Magnetization of Hard Superconductors
C.P. Bean, Phys. Rev. Lett. 8, 250 (1962)

The basic premise of this theory\textsuperscript{10,11} is that there exists a limiting macroscopic superconducting current density \( J_c(H) \) that a hard superconductor can carry; and further, that any electromotive force, however small, will induce this full current to flow
The critical state > 50 years!

Magnetization of Hard Superconductors
C.P. Bean, Phys. Rev. Lett. 8, 250 (1962)

citations: 2785

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\[ \mu_0 j = \nabla \times B \]
Normal Superconductor

Faraday:

$$\nabla \times \vec{B} = \mu_0 (\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t})$$

$$\vec{B} = B_z(x) \hat{z}$$

$$\nabla \times \vec{B} = - \frac{\partial B_z}{\partial x} \hat{y} = \mu_0 \dot{J}$$

Critical State: $\vec{J} = J_c$

Bean Model: $J_c$ const.
Vortices are present in almost all applications of superconductors;

Vortices have a dynamics of their own;

This dynamics determines the superconducting properties which are relevant for applications.
II. Vortices
Vortices in Nature
Vortices in Nature

Normal fluids:
- viscosity
- “rigid body” rotation

Superfluids:
- no viscosity
- vortices
Vortices in Nature

Bose Condensate: superfluid He4
Vortices in Nature

Bose Condensate: cold atoms

WHEN ATOMS BEHAVE AS WAVES: BOSE-EINSTEIN CONDENSATION AND THE ATOM LASER

Nobel Lecture, December 8, 2001
by
WOLFGANG KETTERLE*

Department of Physics, MIT-Harvard Center for Ultracold Atoms, and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts, 02139, USA.

Figure 20. Observation of vortex lattices in rotating Bose-Einstein condensates. The examples shown contain (A) 16 (B) 32 (C) 80 and (D) 130 vortices as the speed of rotation was increased. The vortices have "crystallized" in a triangular pattern. The diameter of the cloud in (D) was 1 mm after ballistic expansion, which represents a magnification of twenty. (Reprinted with permission from ref. [112]. Copyright 2001 American Association for the Advancement of Science.)
Vortices in Superconductors
Vortices in Superconductors

Abrikosov lattice

Magneto-optical Imaging

Tom H. Johansen

http://www.fys.uio.no/super/
1957 - Abrikosov predicted the existence of type II SCs (flux allowed)

Surface energy can be negative in certain cases \( \kappa = \frac{\lambda}{\xi} > 0.707 \)

\( \Rightarrow \) creation of interfaces N/SC become energetically favorable

Fluxoids or Vortices: normal regions in the form of tubes carrying one flux quantum each, \( \phi_0 \sim 2 \times 10^{-15} \) SI, surrounded by screening currents.
Vortex matter

\[ \int B \, dA = \frac{\hbar}{2e} = \Phi_0 \]

Flux quantum
Length scales in superconductivity

Ginzburg-Landau Theory:

\[ G_s(\phi, \vec{A}) = \]
\[ = G_n + \frac{1}{V} \int d^3 \vec{r} \left[ \frac{1}{2m^*} \hat{p}^* \phi^* \cdot \hat{p} \phi + \frac{B^2(\vec{r})}{2\mu_0^2} - \mu_0 \vec{H}(\vec{r}) \cdot \vec{M}(\vec{r}) \right] \]
\[ + a\phi\phi^* + \frac{b}{2} \phi\phi^*\phi\phi^* + \ldots \]

where \( \hat{p} = i\hbar \hat{\nabla} + e^* \hat{A} \) is the canonical moment and the coefficients are taken as

\[ a(T) \sim a_0 \left[ \frac{T}{T_c} - 1 \right]; \quad b(T) \sim b_0; \quad T \sim T_c \]
Length scales in superconductivity

**Ginzburg-Landau Theory:**

\[ G_s(\phi, A) \]

\[ \delta_\phi G_s = 0 \text{ e } \delta_A G_s = 0 \rightarrow 2 \text{ GL equations} \]

**Dimensionless GL equations**

\[ \lambda : \text{ space scale in equation arising from } \delta_A G_s = 0 \]

\[ \xi : \text{ space scale in equation arising from } \delta_\phi G_s = 0 \]

**Energy associated to formation of N/S interface:**

\[ -\sigma_{NS} \alpha (\xi - \sqrt{2} \lambda) \]

\[ -\kappa = \lambda/\xi : \text{ parâmetro de GL} \]
Ginzburg-Landau Equations:

\[
\frac{1}{2m^*}\left(\hbar^2 \nabla^2 \phi - 2i\hbar e^* A \cdot \nabla \phi - e^* A^2 \phi\right) - a\phi - b|\phi|^2 \phi = 0. \quad (6.8)
\]

\[
\nabla \times (\nabla \times A) + \frac{i\hbar e^*}{2m^*} (\phi^* \nabla \phi - \phi \nabla \phi^*)
+ \frac{e^*}{m^*} A|\phi|^2 = 0. \quad (6.10)
\]

From Ampère’s Law:

\[
\mu_0 J = -\frac{i\hbar e^*}{2m^*} (\phi^* \nabla \phi - \nabla \phi^* \phi) - \frac{e^*}{m^*} A|\phi|^2. \quad (6.12)
\]

Superconductivity; Poole, Farach, Creswick and Prozorov
Vortex Quantization

\[ \phi(\mathbf{r}) = |\phi(\mathbf{r})|e^{i\Theta} \]  \hspace{1cm} (6.2)

\[ \nabla \phi = i\phi \nabla \Theta + e^{i\Theta} \nabla |\phi(\mathbf{r})|, \]  \hspace{1cm} (6.31)

\[ \frac{m^*}{e^*^2} \int \frac{\mu_0 \mathbf{J}}{|\phi|^2} \cdot d\mathbf{l} + \Phi = n\Phi_0. \]  \hspace{1cm} (6.39)

\[ \Phi_0 = \frac{h}{e^*}, \]  \hspace{1cm} (6.36)
First Image
Bitter Decoration 1967
Pb-4at%In rod, 1.1K, 195G
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Vortex Dynamics
Vortices in the presence of currents: viscous motion $\Rightarrow$ dissipation
Vortices (fluxoids) carry quantized flux, \( \Phi = n \Phi_0 \) (usually \( n = 1 \))

Vortices in the presence of currents: viscous motion \( \rightarrow \) dissipation
Vortices in the presence of currents: viscous motion $\Rightarrow$ dissipation

- Vortices (fluxoids) carry quantized flux, $\Phi = n \Phi_0$ (usually $n = 1$)
- Collection of vortices: typical elastic, electric, magnetic & thermal properties $\Rightarrow$ Vortex Matter (VM)
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- Collection of vortices: typical elastic, electric, magnetic & thermal properties \( \rightarrow \) Vortex Matter (VM)
- If \( J \) is present, VM experiences viscous movement which may lead the sample to its normal state
Vortices (fluxoids) carry quantized flux, $\Phi = n \Phi_0$ (usually $n = 1$)

- Collection of vortices: typical elastic, electric, magnetic & thermal properties $\Rightarrow$ Vortex Matter (VM)

- If $J$ is present, VM experiences viscous movement which may lead the sample to its normal state

- Pinning centers (PC) can prevent such movement, trapping vortices in potential wells

- PCs are crucial to enable $J_c > 0$
Alexei Abrikosov acting as a “pinning center” for his admirers
Leuven, July 2006
VORTICES IN SCs: BASICS

- Vortex entry

\[ F_L = J \times \Phi_0 \]
Interactions

vortex-vortex

\( \Phi_0 \) \( \Phi_0 \)

vortex-current

\[ j \]

\[ \vec{f}_L = \vec{j} \times \Phi_0 \]

\( E \propto dB / dt \)

Loss = \( jE \)

pinning

\[ f_p \]

\[ f_L \]

\[ k_B T \]

Loss = 0
defects: pinning centers

\[ \xi \]

\[ \lambda \]

\[ B(r) \]

\[ j \]
Vortex Avalanches
Facts

Under certain conditions of temperature and magnetic field, flux avalanches of dendritic form develop into superconducting films, as a consequence of thermomagnetic instabilities (TMI);
Magneto optical images of avalanches in superconducting thin films

- **Nb**
  - 5.97 K, 135 Oe
  - Remanent state
  - Durán et al. 1995

- **Nb₃Sn**
  - 3.5 K, 263 Oe
  - Rudnev et al. 2003

- **YBCO**
  - 1.8 K, 600 Oe
  - After laser pulse
  - Leiderer et al. 2004

- **MgB₂**
  - 10 K, 170 Oe
  - Johansen et al. 2004

- **NbN**
  - 4.8 K, 26 Oe
  - Rudnev et al. 2005

- **Nb**
  - 5.97 K, 135 Oe
  - Welling et al. 2004
Some images captured @ GSM/São Carlos

- **Nb (Plain)**
  - Remanent state
  - 3 K, after 4 Oe
  - GSM, 2011

- **α-MoSi (Plain)**
  - 3 K
  - ZFC → 60 Oe → 10 Oe
  - GSM, 2011

- **α-MoGe (AD04)**
  - 4.5 K, 1 Oe
  - GSM, 2011
α-MoGe (AD04); 4.5 K, 1 Oe

GSM, 2011
Thermomagnetic Instabilities

Thermal diffusion ➞ Magnetic diffusion

Magnetic field penetrates smoothly
Reentrant stability of superconducting films and the vanishing of dendritic flux instability

V. V. Yurchenko, D. V. Shantsev, and T. H. Johansen

\[
H^\text{th} = \frac{j_c d}{\pi} \arccosh\left(\frac{w}{w - \ell^*}\right)
\]

\[
\ell^* = \frac{\pi}{2} \sqrt{\frac{\kappa T^*}{j_c E}} \left(1 - \sqrt{\frac{2 h_0 T^*}{n d j_c E}}\right)^{-1},
\]

provided that \(\ell^* < w\). Here, \(j_c\) is the critical current density, \(T^* = -(\partial \ln j_c / \partial T)^{-1}\), \(E\) is the electric field, \(\kappa\) is the thermal conductivity, and \(h_0\) is the coefficient of heat transfer from the superconducting film to the substrate. The parameter \(n\) characterizes the nonlinearity of the current-voltage curve of
Magnetic field, $H$

Critical current density, $j_c$

$H^{th}_{2}$

$H^{th}_{1}$

$H^{th}(j_c)$

$H^{th}_{2}$

$H^{th}_{1}$

$TMI$ region

$H_{c1}$

$H_{c2}$

 instability region

$H^{th}(j_c)$

smaller $j_c(B)$ - always stable
Granular matter $\Rightarrow$ avalanches

& destroy transport abilities
Vortex matter \sim granular medium

P. G. DeGennes:
“We can get some physical feeling of this critical state by thinking of a sand hill”
book on Superconductivity (1966)

Expect:
Complex dynamics
Critical Current Threshold

- $J_c$ vs. $H$ (Oe)
- $J_c$ before the 1st avalanche
- Average $J_c$
- Standard Deviation of $J_c$

Figure 13.3: Dependence of the internal magnetic field $B_0(x)$, current density $J_c(x)$, and pinning force $F_p(x)$ on the strength of the applied magnetic field $B_0$ for normalized applied fields given by $B_0 = \frac{1}{2} \mu_0 J_c a$, $B_0 = \mu_0 J_c a$, and $B_0 = 2 \mu_0 J_c a$. This and subsequent figures are drawn for the Bean model. There is a field free region in the center for case (a), while case (b) represents the boundary between the presence versus the absence of such a region.
Reentrant stability of superconducting films and the vanishing of dendritic flux instability

V. V. Yurchenko, D. V. Shantsev, and T. H. Johansen

\[ H^{th} = \frac{j_c d}{\pi} \arccosh \left( \frac{w}{w - \ell^*} \right) \]

\[ \ell^* = \frac{\pi}{2} \sqrt{\frac{\kappa T^*}{j_c E}} \left( 1 - \sqrt{\frac{2h_0 T^*}{nd_j E}} \right) \]

provided that \( \ell^* < w \). Here, \( j_c \) is the critical current density, \( T^* = -\frac{1}{\kappa} \ln \left( \frac{j_c}{\partial j_c / \partial T} \right) \), \( E \) is the electric field, \( \kappa \) is the thermal conductivity, and \( h_0 \) is the coefficient of heat transfer from the superconducting film to the substrate. The parameter \( n \) characterizes the nonlinearity of the current-voltage curve of...
Linearized theory predicts

First finger forms at penetration depth:

\[ l_{\text{th}} = \frac{\pi}{2} \sqrt{\frac{\kappa T^*}{j_c E}} \left( 1 - \sqrt{\frac{2 \hbar T^*}{nd j_c E}} \right)^{-1} \]

…provided \( 2l_{\text{th}} < 2w \)

Threshold field:

\[ H_{\text{th}} = \frac{j_c d}{\pi} \text{arccosh} \left( \frac{w}{w - l_{\text{th}}} \right) \]

\( H_{\text{th}} = \frac{j_c d}{\pi} \text{arccosh} \left( \frac{w}{w - l_{\text{th}}} \right) \)

\( T = 4 \text{ K} \)

\( \mu_0 H_a = 15 \text{ mT} \)

\( 2w = 1.6 \text{ mm} \)

\( 2w = 0.2 \text{ mm} \)

\( 82 \mu\text{m} \)
Experiment - MgB$_2$

Parameters: (MgB$_2$)

- $c = 34 \text{ kJ/Km}^3 \times (T/T_c)^3$
- $\kappa = 172 \text{ W/Km} \times (T/T_c)^3$
- $b = 46 \text{ kW/Km}^2 \times (T/T_c)^3$
- $T_c = 39 \text{ K}$

- $\rho_n = 6.8 \mu\Omega\text{cm}$
- $\dot{\iota} = 10^{-5} J_{c0} \rho_n/\mu d$
- $J_{c0} = 54 \text{ kA/m}$
- $n = 19$
Magneto-optical Imaging (MOI)

A powerful tool to see magnetism and superconductivity in action
Faraday effect: rotation of the polarization plane
Magneto-optical Imaging

- Faraday rotation of polarized light passing through an indicator (with in-plane magnetization), placed in close contact with the SC sample of interest

⇒ space distribution of magnetic flux
Bismuth-substituted Yttrium-Iron garnet

Bi:YIG ($Y_{3-x}Bi_xFe_5O_{12}$)
on (100) substrate of $Gd_3Ga_5O_{12}$ (GGG)

Gadolinium-Galium garnet
MOI setup

MOI setup in São Carlos
Revealing the intrinsic beauty of the problem

Visualizing Magnetic Fields in Superconductors
Intrinsic beauty of the problem
1st Prize: "Photography – Science and Arts"
Brazilian National Research Council (CNPq)

**Category:** Photomicrography – special lenses, microscopes

**Title:**
Visualizing Magnetic Fields in Superconductors

Image recorded by
W. A. Ortiz and coworkers
Univ. Federal de São Carlos,
SP, Brazil.

**Shows:** Magneto-optical image of magnetic flux penetrating into a superconducting film patterned with a square lattice of antidots (nanosized holes not directly visible)

Prize awarded at the opening ceremony of the "National Week on Science and Technology", in Brasília, Oct. 18, 2011.
Visualizando o Campo Magnético em Supercondutores
Primeiro Lugar – Categoria Micro
Celebration in Oslo
Centre for Advanced Study - Norwegian Academy of Science
CNPq – 2013 – 3rd place
$\text{MgB}_2$

$T = 9 \text{ K}$