

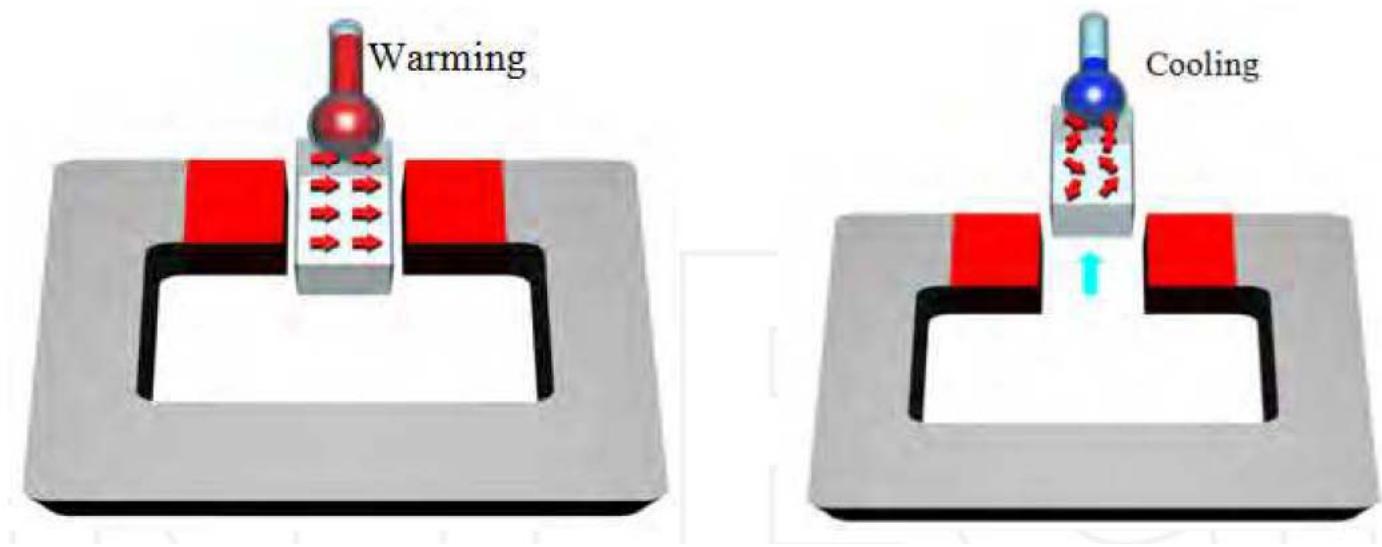
acoustic detection of the magnetocaloric effect

antonio manoel mansanares

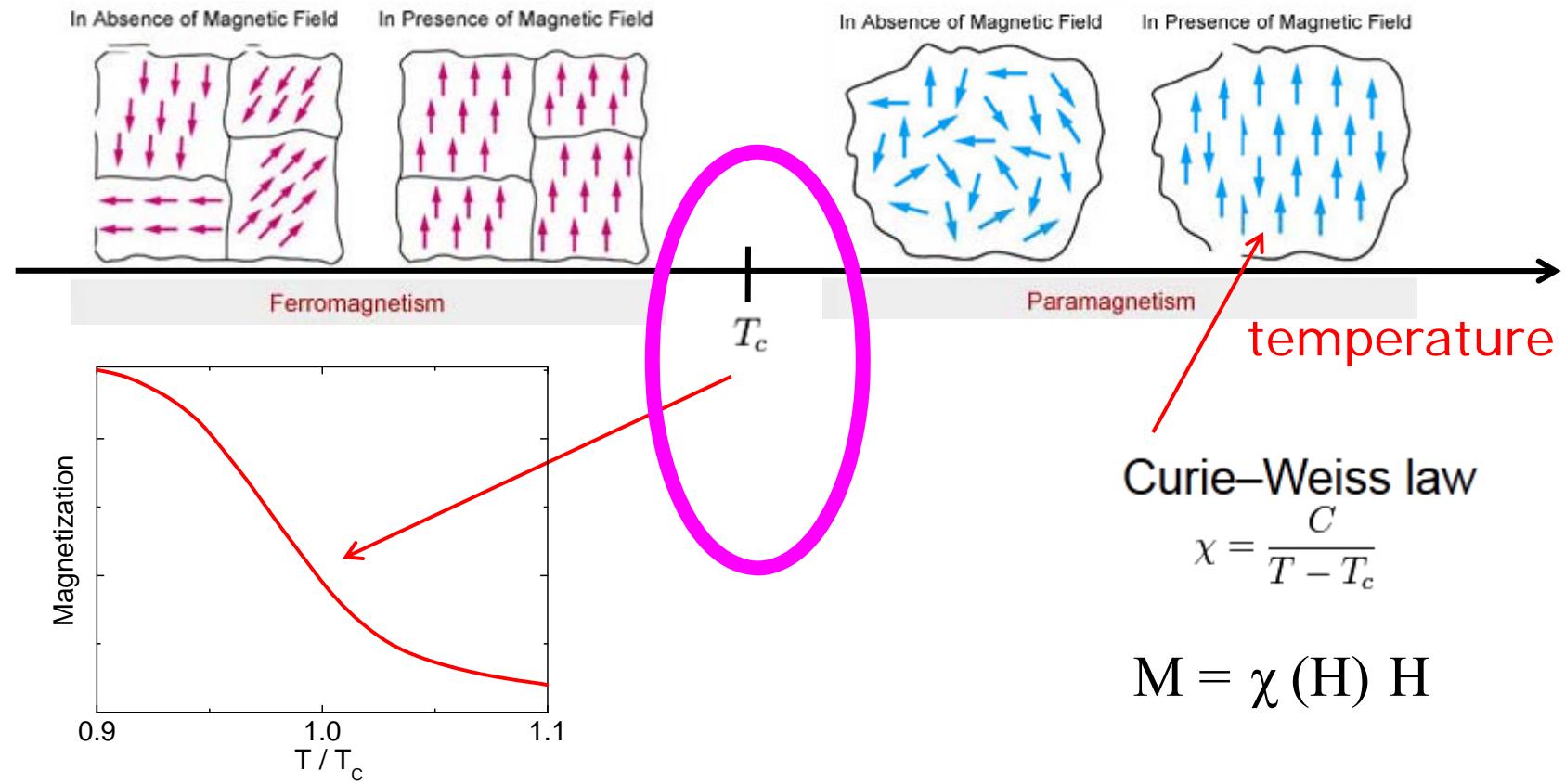
gleb wataghin physics institute, ifgw
campinas state university, unicamp

2015 ifgw winter school

the magnetocaloric effect: what is it?



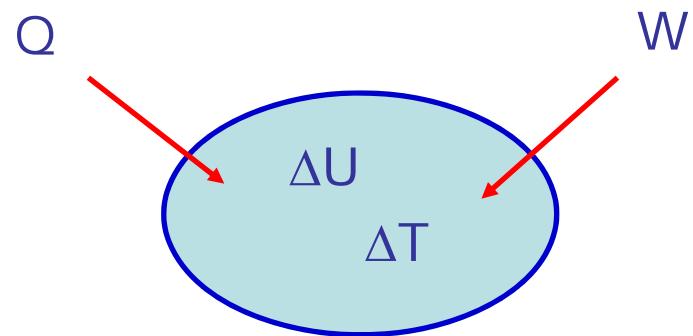
the magnetocaloric effect: strategic/ideal conditions



objectives of the presentation

- to introduce the magnetocaloric effect (mce) and its potential applications: magnetic refrigeration;
- to establish thermodynamic relations between magnetic field, temperature and entropy; discuss the conventional characterization of the mce;
- to introduce the photoacoustic effect: detection of small changes in the temperature of the sample;
- to use the acoustic detection to determine the mce;

the first law of thermodynamics: internal energy U

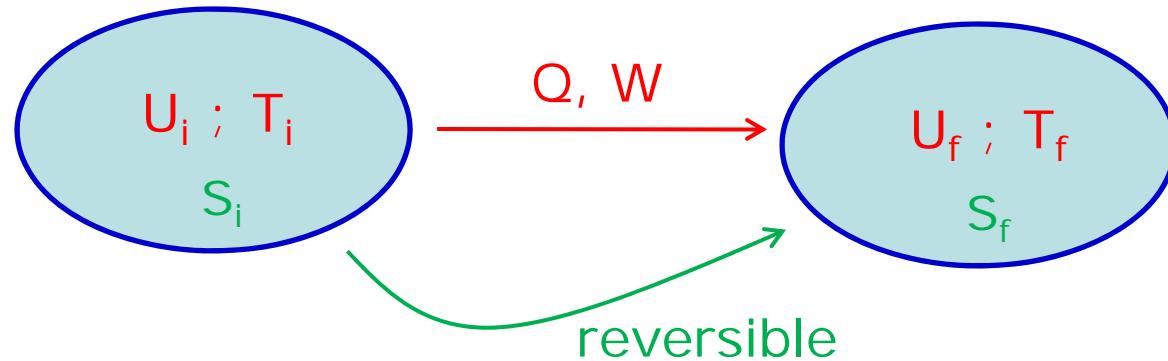


the zero law
states the existence of the
temperature

$$\left\{ \begin{array}{l} \Delta U = Q + W \\ dU = \delta Q + \delta W \end{array} \right.$$

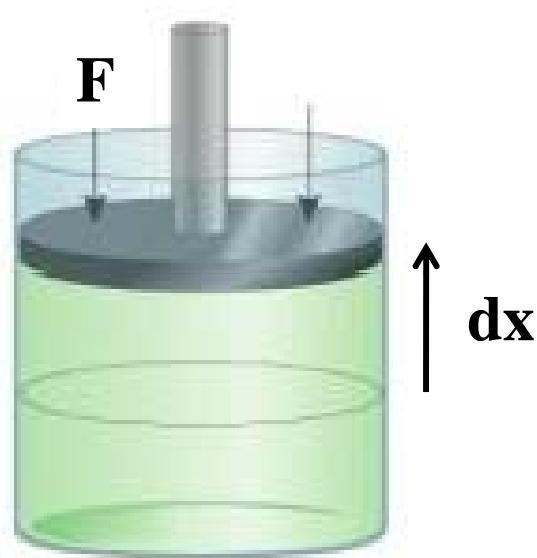
$$\left\{ \begin{array}{l} \text{If } \delta W = 0, \text{ then:} \\ dU = C_{V, M, \dots} dT \\ \uparrow \\ \text{extensive coordinates:} \\ \text{volume, magnetization etc.} \end{array} \right.$$

the second law of thermodynamics: entropy S



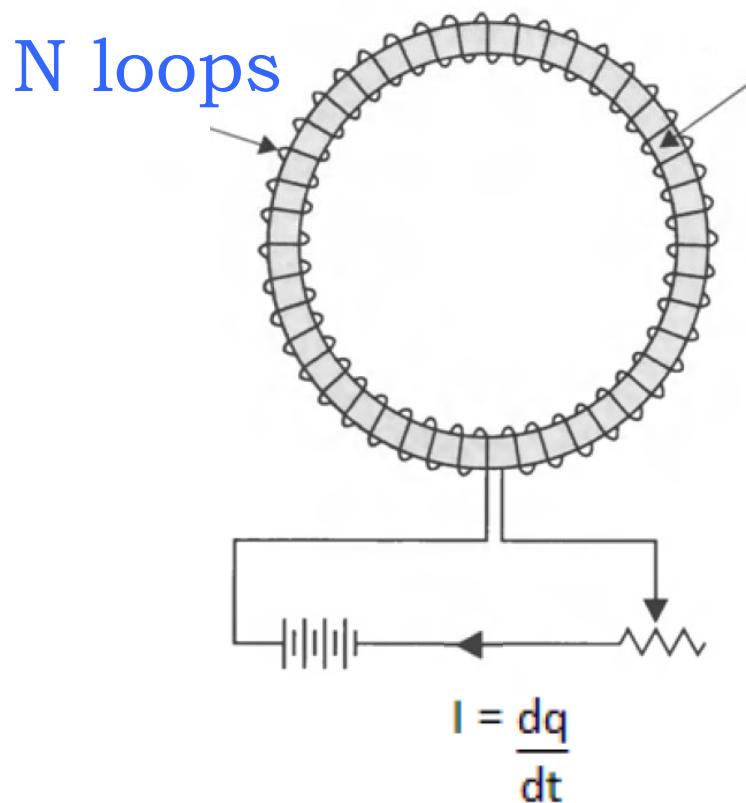
$$\left\{ \begin{array}{l} dS = \frac{\delta Q_{\text{reversible}}}{T} \\ \Delta S = S_f - S_i = \int\limits_i^f \frac{\delta Q}{T} \\ \text{reversible path} \end{array} \right.$$

work on a hydrostatic system:
a gas, for example



$$\delta W = - F dx = - P A dx = - P dV$$

work on a magnetic system



magnetic material

$$\varepsilon = - \frac{1}{c} N \frac{\partial \Phi}{\partial t} \text{ (gaussian units)}$$

$$\Phi = \oint \vec{B} \cdot \hat{n} da = BA \text{ (thin toroid)}$$

$$\varepsilon = - \frac{1}{c} NA \frac{dB}{dt}$$

work on a magnetic system

work done by the battery: $\delta W' = -\varepsilon dq = -\varepsilon I dt = \frac{NAIdB}{c}$

Ampère law: $\oint_C \vec{H} \cdot d\vec{l} = \frac{4\pi}{c} I_{enc} \Rightarrow H = \frac{4\pi NI}{cL} \Rightarrow (NI) = \frac{HLc}{4\pi}$

$$\delta W' = \frac{(NI)AdB}{c} = \frac{HLc}{4\pi} \frac{AdB}{c} = \frac{VH}{4\pi} dB$$

$$B = H + 4\pi M \Rightarrow dB = dH + 4\pi dM$$

$$\delta W' = \frac{VH}{4\pi} dH + V H dM$$

vacuum material

magnetization



work on a magnetic system

$$\delta W = V H dM = H dm$$



total magnetic
moment

$$\vec{m} = V \vec{M} = \sum_i \vec{m}_i$$

a magnetic system: the first law of thermodynamics

$$dU = \delta Q - PdV + H dm$$

↑
mechanical
work

← magnetic
work

practical conditions:

- i) atmospheric pressure ($p = \text{cte}$)
- ii) solids: volume $\sim \text{cte}$

Therefore, p and V are taken as constants:

$$dU = \delta Q + H dm$$

the magnetocaloric effect: the formal concept

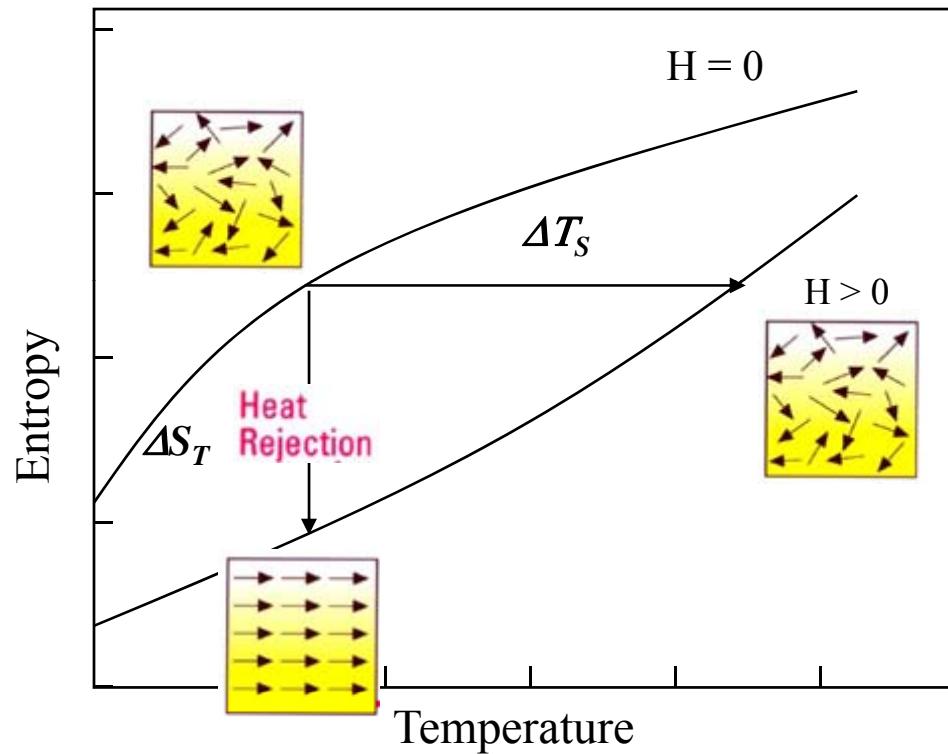
$$dU = \delta Q + H dm$$

↑
work

$$\delta Q = 0 \text{ (adiabatic)}$$
$$H \uparrow \quad T \uparrow \quad (\Delta T_s > 0)$$

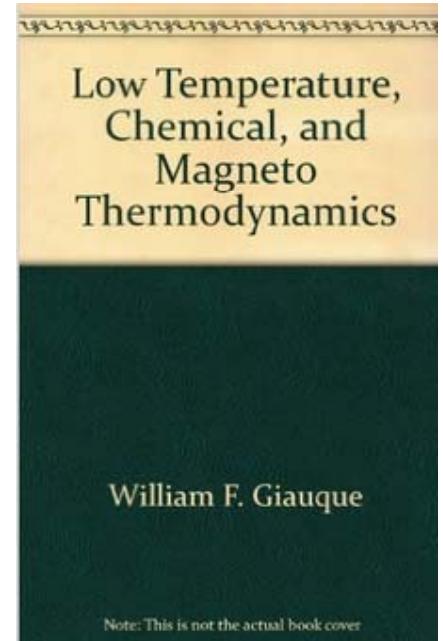
$$T = \text{cte} \text{ (isothermal)}$$
$$H \uparrow \quad \delta Q = T dS \downarrow \quad (\Delta S_T < 0)$$

the magnetocaloric effect



one slide of history

- 1881: discovery by the German physicist Emil Gabriel Warburg. He observed a few millikelvins increase in an iron sample [1].
- 1918: Weiss and Piccard explained the effect [2].
- 1926/1927: Debye [3] and Giauque [4] proposed a new method to reach ultra low temperatures (below 1 K) using adiabatic demagnetization.
- 1933: experimental proof of the method by Giauque [5]. A temperature of 0.25 K was attained, and the use of the effect in magnetic refrigeration at low temperatures became important.
William Francis Giauque was an American chemist and Nobel laureate recognized in 1949 for his studies in the properties of matter at temperatures close to absolute zero.
- 1976: Brown demonstrated the usefulness of the method at room temperature [6]. He obtained a temperature variation of 47 K by cycling Gd following a suitable pathway. The maximum mce in Gd at 7 T is about 16 K.
- 1997: Pecharsky and Gschneidner discovered a new alloy ($\text{Gd}_5\text{Si}_2\text{Ge}_2$) that presents a giant mce near room temperature (~ twice the pure Gd) [7].

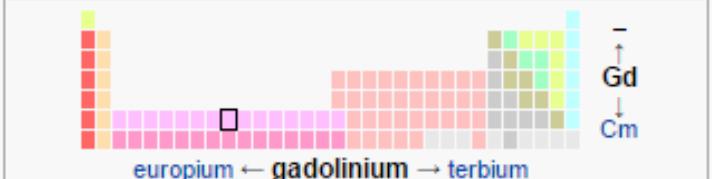


history key references

1. E. Warburg, *Magnetische Untersuchungen über einige Wirkungen der Koerzitivkraft*, Ann. Phys. **13** (1881) 141–164.
2. P. Weiss, A. Piccard, *Sur un nouveau phénomène magnétocalorique*, Compte Rendu Ac. Sci. **166** (1918) 352.
3. P. Debye, *Einige Bemerkungen zur Magnetisierung bei tiefer Temperatur*, Ann. Phys. **81** (1926) 1154–1160.
4. W.F. Giauque, *A thermodynamic treatment of certain magnetic effects. A proposed method of producing temperatures considerably below 1° absolute*, J. Am. Chem. Soc. **49** (1927) 1864–1870.
5. W.F. Giauque, D.P. MacDougall, *Attainment of temperatures below 1° absolute by demagnetization of $Gd_2(SO_4)_3 \cdot 8H_2O$* , Phys. Rev. Lett. **43** (9) (1933) 768.
6. G.V. Brown, *Magnetic heat pumping near room temperature*, J. Appl. Phys. **47** (1976) 3673–3680.
7. V. K. Pecharsky, K. A. Gschneidner Jr., *Giant Magnetocaloric effect in $Gd_5(Si_2Ge_2)$* , Phys Rev Lett **78** (23) (1997) 4494–4497.

gadolinium



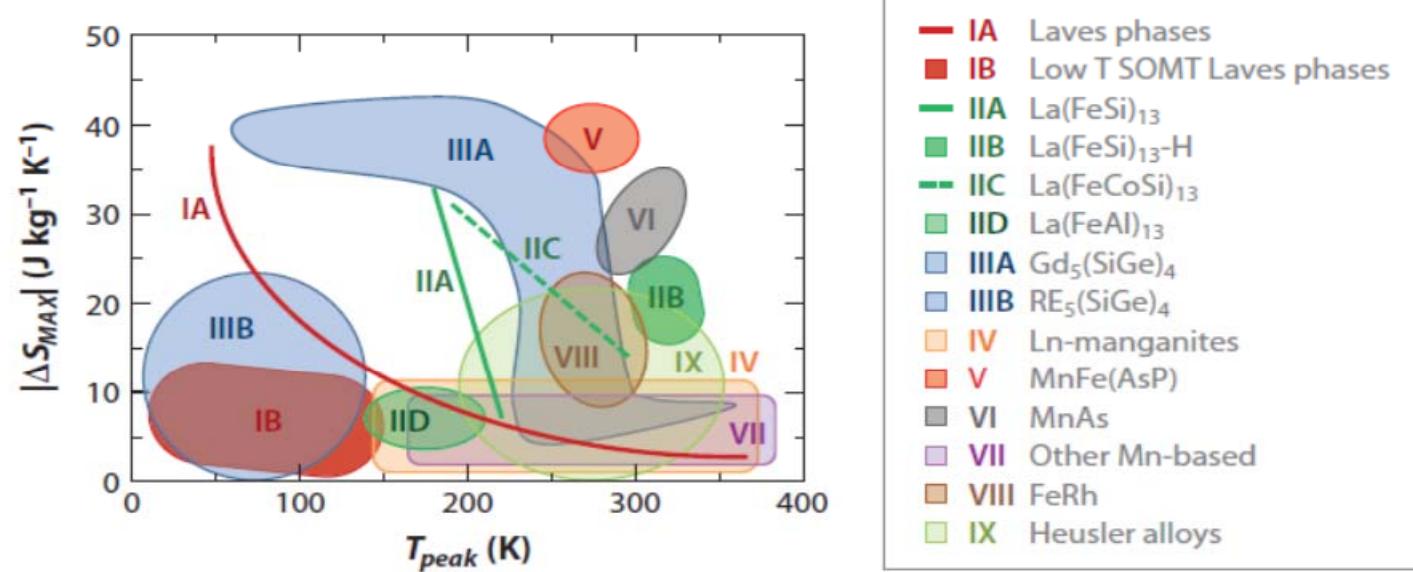
Gadolinium in the periodic table	
	Gd Cm
Atomic number	64
Standard atomic weight (\pm) (A_r)	157.25(3) ^[1]
Element category	lanthanide
Group, block	group n/a, f-block
Period	period 6
Electron configuration	[Xe] 4f ⁷ 5d ¹ 6s ²
per shell	2, 8, 18, 25, 9, 2
Physical properties	
Phase	solid
Melting point	1585 K (1312 °C, 2394 °F)
Boiling point	3273 K (3000 °C, 5432 °F)
Density near r.t.	7.90 g/cm ³
when liquid, at m.p.	7.4 g/cm ³
Heat of fusion	10.05 kJ/mol
Heat of vaporization	301.3 kJ/mol
Molar heat capacity	37.03 J/(mol·K)

Miscellanea	
Crystal structure	hexagonal close-packed (hcp)
	
<u>Speed of sound</u> thin rod	2680 m/s (at 20 °C)
Thermal expansion	α poly: 9.4 $\mu\text{m}/(\text{m}\cdot\text{K})$ (at 100 °C)
Thermal conductivity	10.6 W/(m·K)
Electrical resistivity	α , poly: 1.310 $\mu\Omega\cdot\text{m}$
Magnetic ordering	ferromagnetic-paramagnetic transition at 293.4 K
Young's modulus	α form: 54.8 GPa
Shear modulus	α form: 21.8 GPa
Bulk modulus	α form: 37.9 GPa
Poisson ratio	α form: 0.259
Vickers hardness	510–950 MPa
CAS Registry Number	7440-54-2
History	
Naming	after the mineral Gadolinite (itself named after Johan Gadolin)
Discovery	Jean Charles Galissard de Marignac (1880)
First isolation	Lecoq de Boisbaudran (1886)

gadolinium



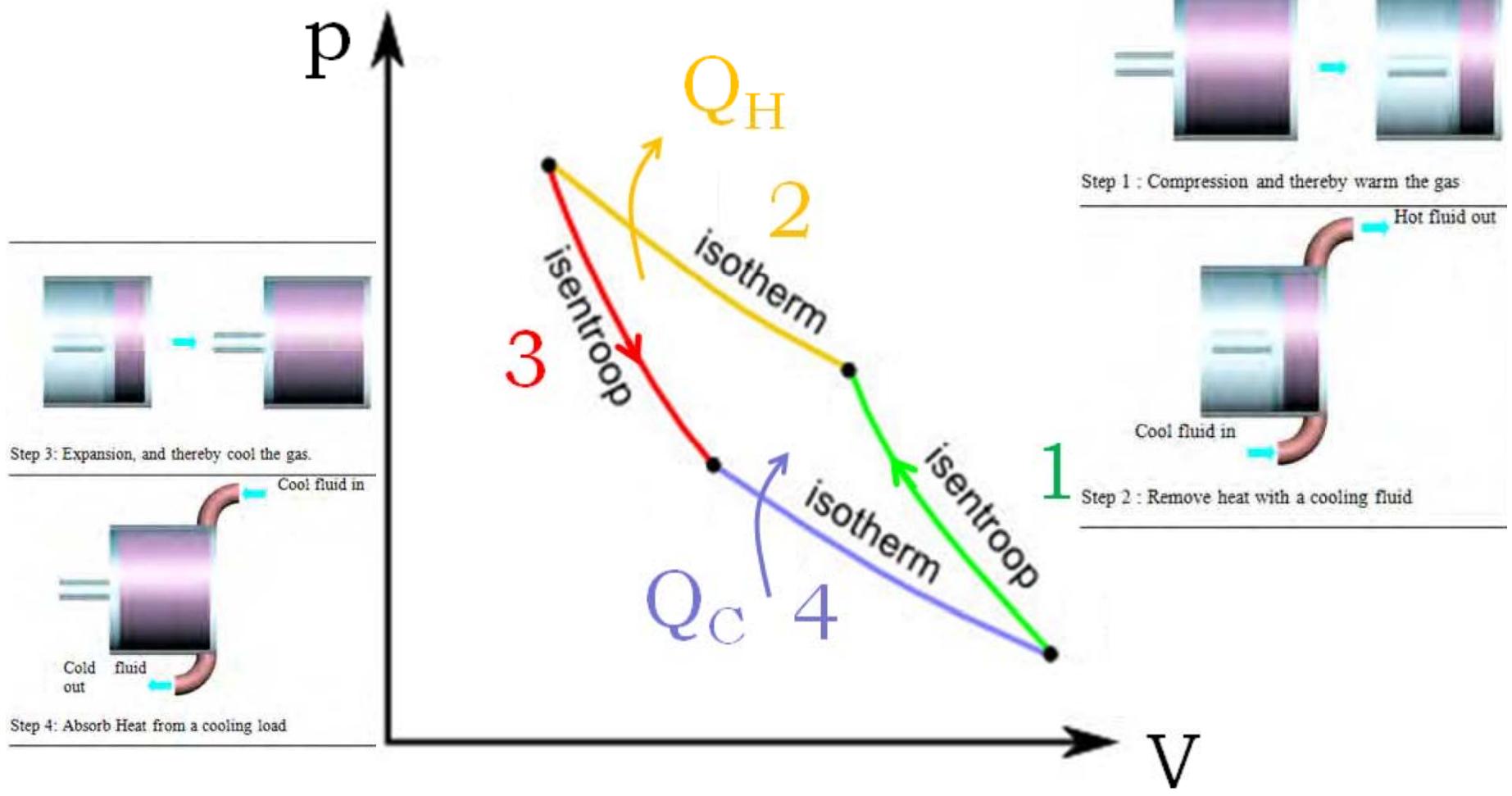
materials



Maximum magnetic entropy change for $\Delta H = 5 \text{ T}$ versus peak temperature for different families of magnetocaloric materials.

The Magnetocaloric Effect and Magnetic Refrigeration Near Room Temperature:
Materials and Models, V. Franco, J.S. Blázquez, B. Ingale, and A. Conde
Annual Review of Materials Research **42**: 305-342 (2012)

conventional refrigeration: ideal gas Carnot cycle

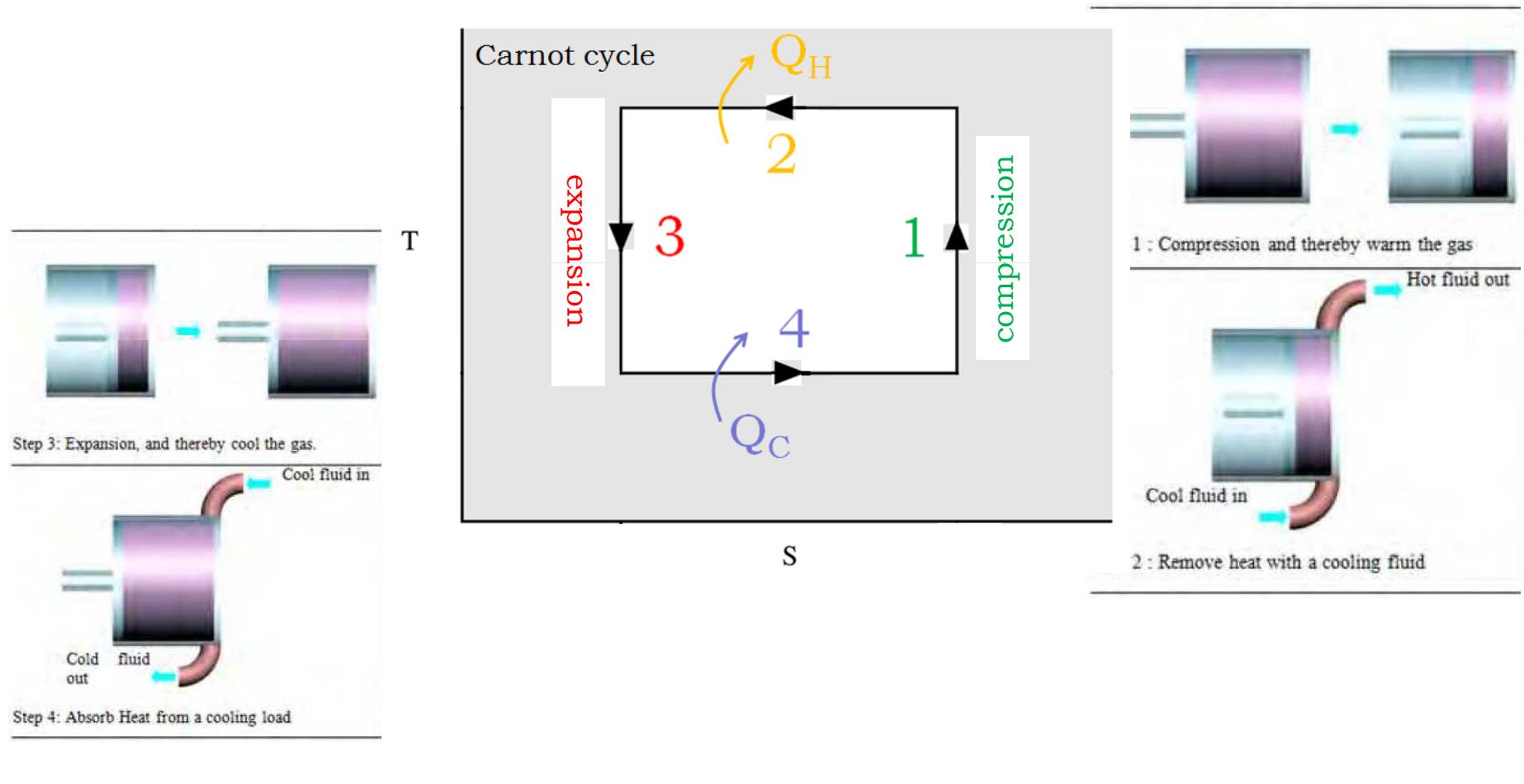


a. m. mansanares

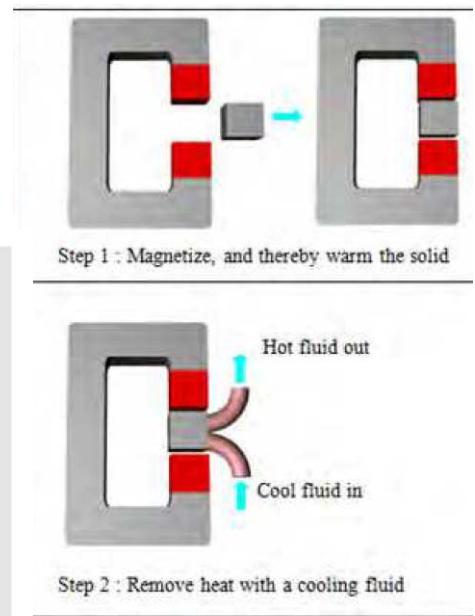
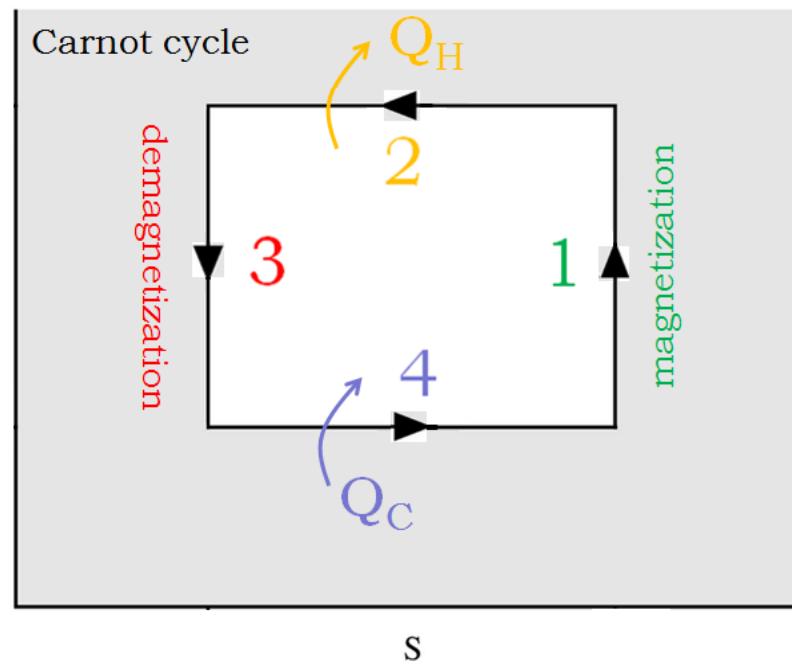
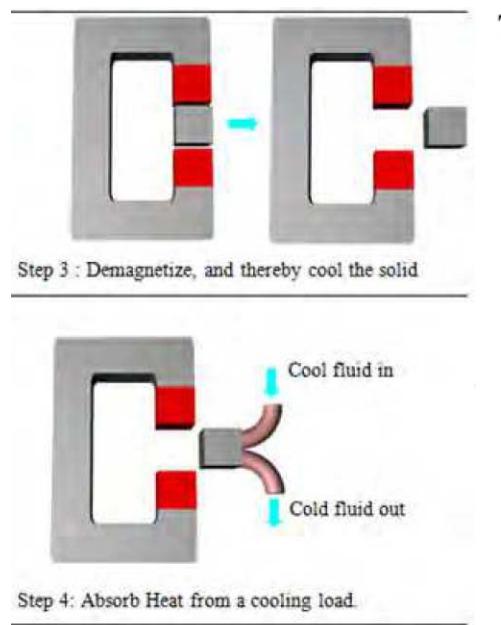
2015 ifgw winter school



conventional refrigeration: ideal gas



magnetic refrigeration



magnetic refrigeration

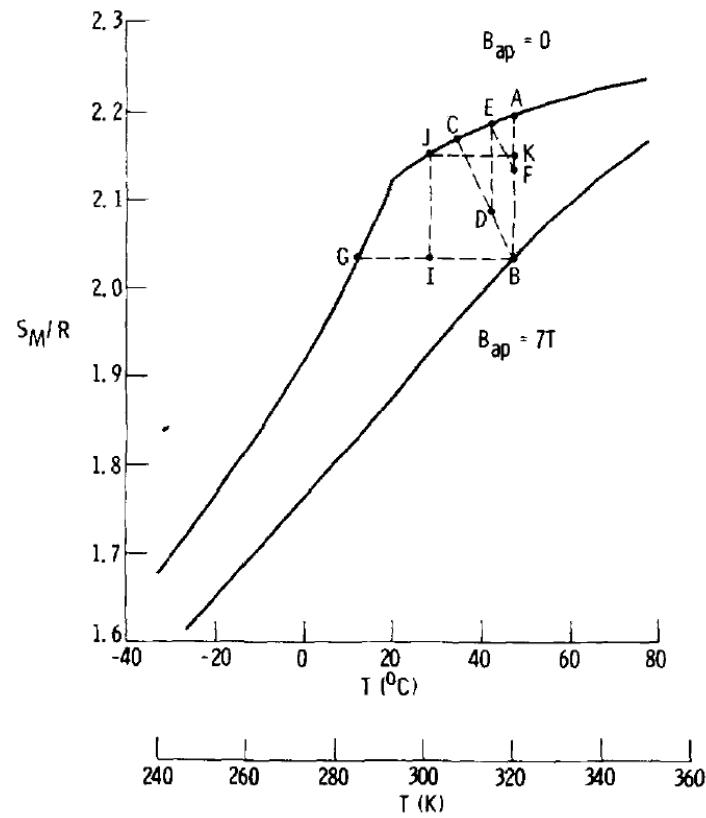


FIG. 6. Magnetic entropy S_M of Gd as a function of temperature T for two applied field strengths B_{ap} .

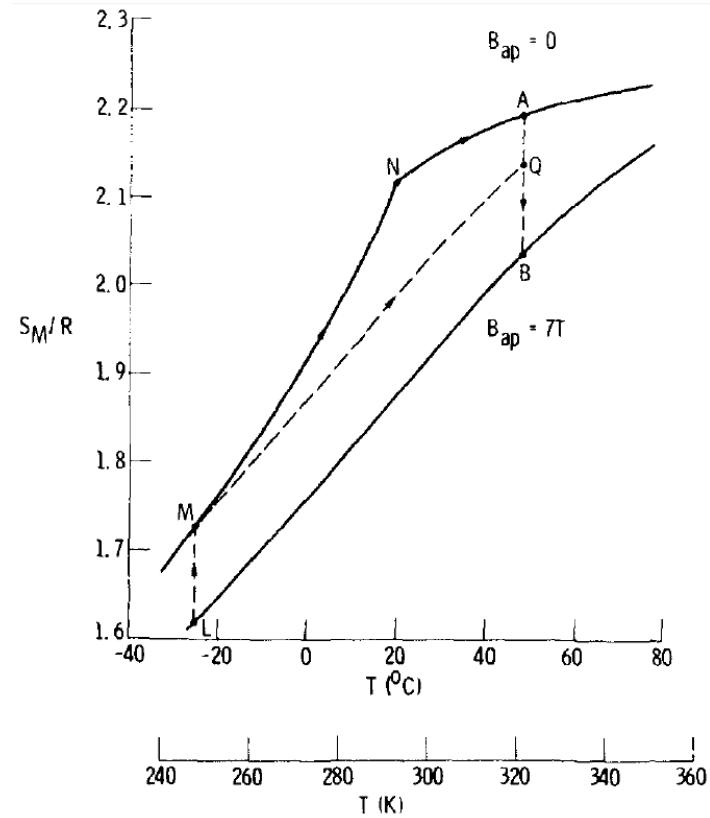
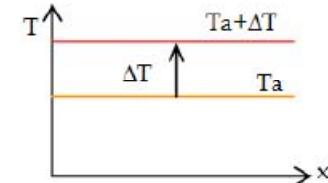
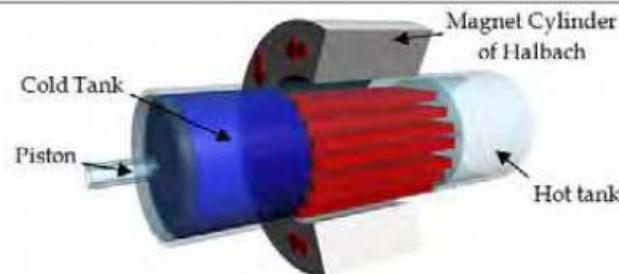


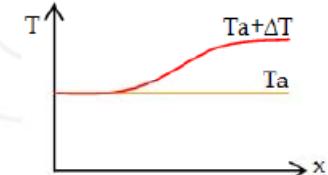
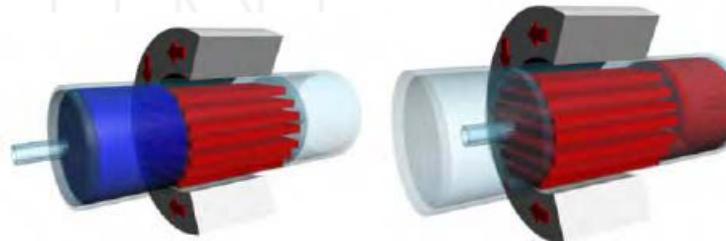
FIG. 8. The magnetic Stirling cycle QBLMQ plotted in the magnetic entropy-temperature plane.

G.V. Brown, J. Appl. Phys. **47** (1976) 3673–3680.

regenerative magnetic refrigeration

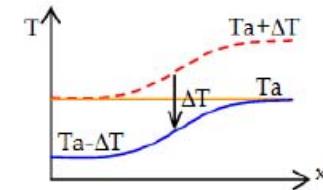
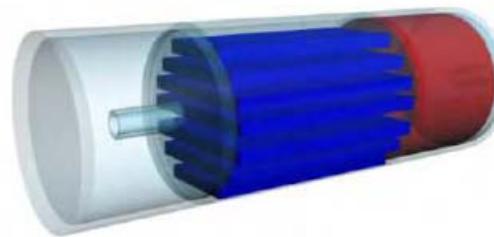


Step 1: Magnetization of the material from an initial state where the entire system is at temperature T_a . Each point of the regenerator material sees its temperature increase by ΔT following the application of the magnetic field.

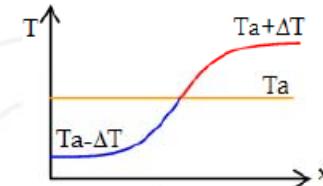
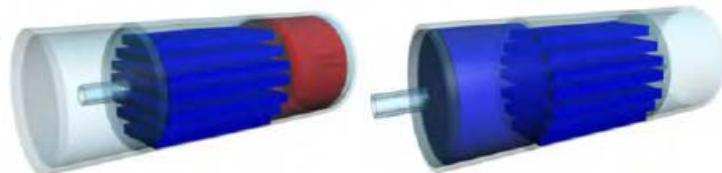


Step 2: Flow of the fluid from the cold source to the hot source. The heat produced by the magnetization step is removed by the fluid flowing from the cold source T_c to the hot source T_h . This creates a temperature gradient along the bed.

regenerative magnetic refrigeration



Step 3: demagnetization of the material. The temperature of Each point of the regenerator decreases by ΔT due to the demagnetization.

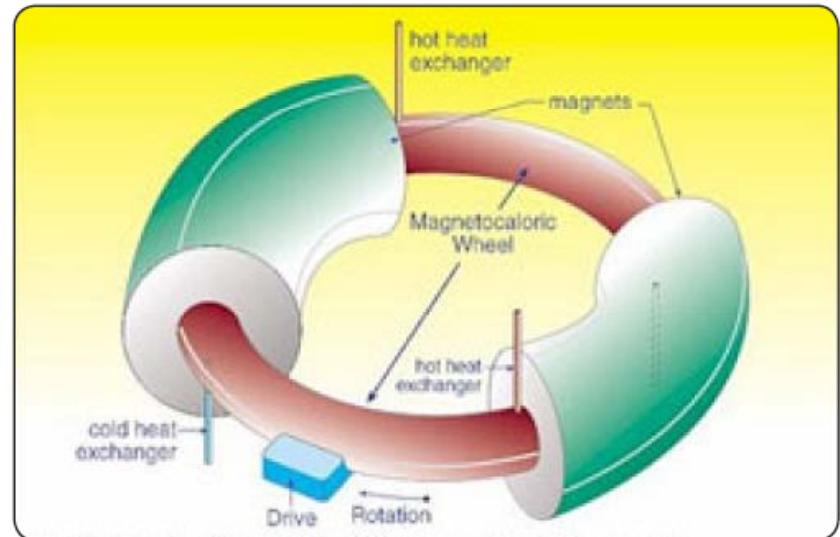
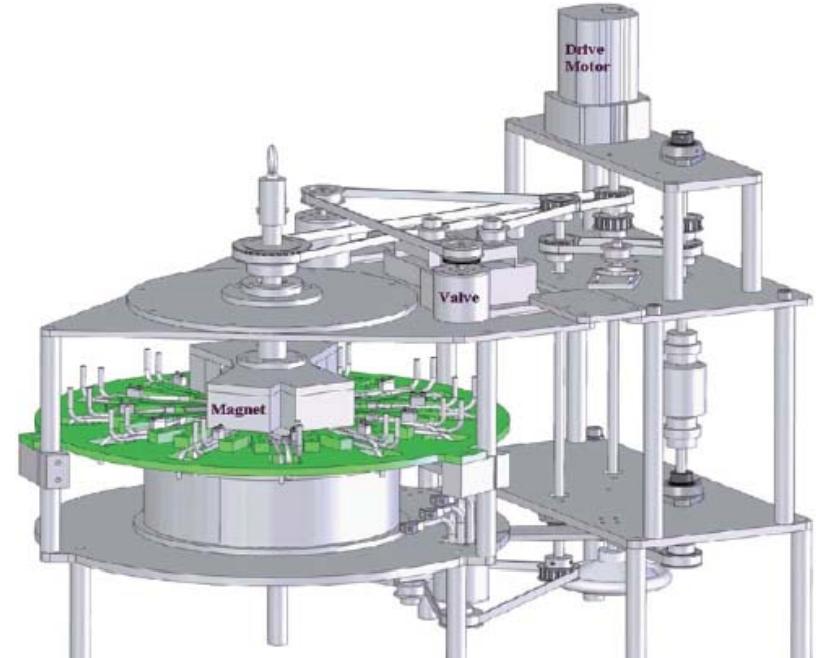


Step 4: Flow of fluid from the hot source to cold source. The flow of the fluid from the hot source T_h to the cold source T_f transfers its heat to the regenerator. The temperature gradient is amplified.

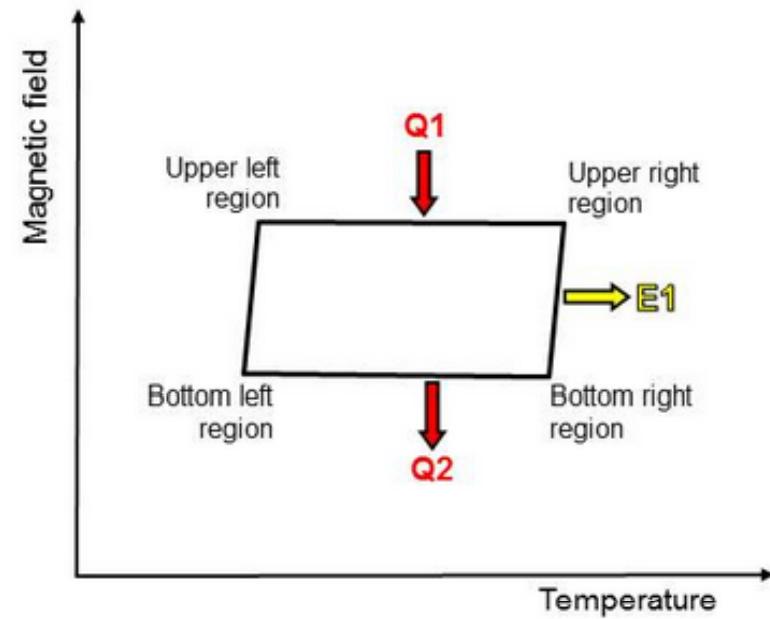
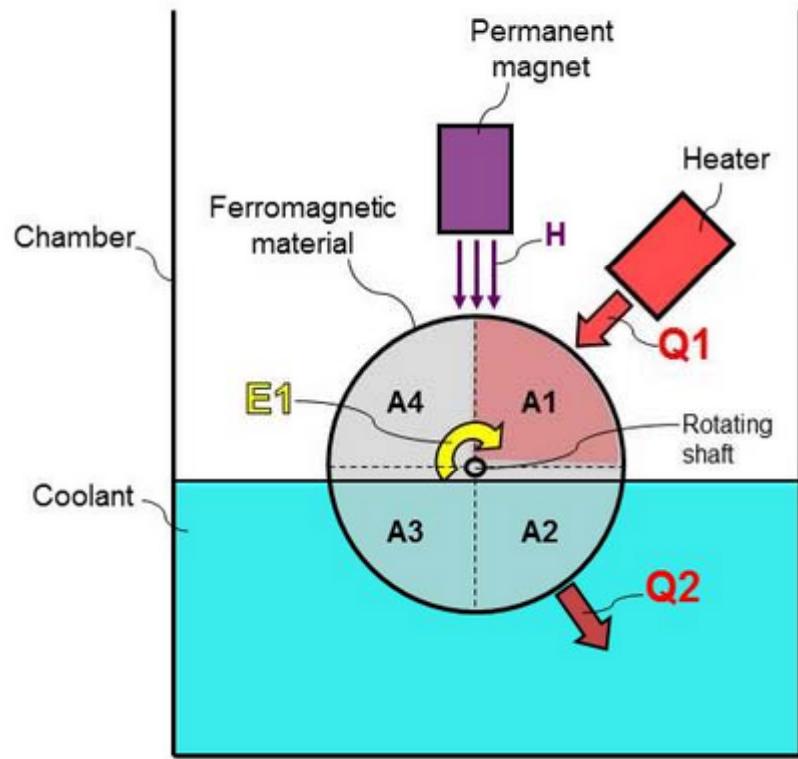
rotating magnetic refrigerator



A rotating magnetic refrigerator developed by Astronautics Corporation of America Ltd. in collaboration with the Ames Laboratory.



thermo magnetic engine



hydrostatic system thermodynamic relations

simple thermodynamic system:

3 coordinates among
T, P, V (the natural coordinates),
U, S and others...

+ 1 equation of state →
→ 2 independent coordinates

ideal gas: $PV = nRT$

$$U = U(T, V)$$

$$dU = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV$$

$$V = \text{cte} \Rightarrow dU = \delta Q = TdS$$

$$\text{and } \left. \frac{\delta Q}{dT} \right|_V = C_V = \left(\frac{\partial U}{\partial T} \right)_V$$

$$\lambda = V \text{ or } P$$

$$C_\lambda = \left. \frac{\delta Q}{dT} \right|_\lambda = \left. \frac{TdS}{dT} \right|_\lambda = \frac{T}{dT} \left[\left(\frac{\partial S}{\partial T} \right)_\lambda dT + \left(\frac{\partial S}{\partial \lambda} \right)_T d\lambda \right]$$

$$\text{For } \lambda = \text{cte}: C_\lambda = T \left(\frac{\partial S}{\partial T} \right)_\lambda$$

$$C_V = T \left(\frac{\partial S}{\partial T} \right)_V \text{ and } C_P = T \left(\frac{\partial S}{\partial T} \right)_P$$

hydrostatic system thermodynamic relations

first law: $dU = dQ + dW$

$$\left\{ \begin{array}{l} dQ = TdS \\ dW = -PdV \end{array} \right.$$

$$dU = TdS - PdV$$

$$U = U(S, V)$$

$$dU = \left(\frac{\partial U}{\partial S} \right)_V dS + \left(\frac{\partial U}{\partial V} \right)_S dV \Rightarrow \left(\frac{\partial U}{\partial S} \right)_V = T \text{ and } \left(\frac{\partial U}{\partial V} \right)_S = -P$$

$$\text{Since } \left(\frac{\partial^2 U}{\partial V \partial S} \right) = \left(\frac{\partial^2 U}{\partial S \partial V} \right) \Rightarrow \left(\frac{\partial T}{\partial V} \right)_S = - \left(\frac{\partial P}{\partial S} \right)_V \text{ (Maxwell equation)}$$

magnetic system

thermodynamic relations

the natural coordinates are: T, H and m

paramagnetic system:

the equation of state is the Curie law: $M = (m/V) = C_C (H/T)$

first law: $dU = TdS + HdM$

$$\text{Maxwell equation: } \left(\frac{\partial T}{\partial M} \right)_S = \left(\frac{\partial H}{\partial S} \right)_M$$

$$C_M = \left(\frac{\partial U}{\partial T} \right)_M = T \left(\frac{\partial S}{\partial T} \right)_M \quad \text{and} \quad C_H = T \left(\frac{\partial S}{\partial H} \right)_H$$

thermodynamic relations

enthalpy E and magnetic enthalpy E^*

$$E = U + PV$$

$$dE = dU + PdV + VdP = TdS + VdP$$

$$P = \text{cte} \Rightarrow dE = \delta Q$$

$$\text{Maxwell equation: } \left(\frac{\partial T}{\partial P} \right)_S = \left(\frac{\partial V}{\partial S} \right)_P$$

$$E^* = U - Hm$$

$$dE^* = dU - Hdm - mdH = TdS - mdH$$

$$H = \text{cte} \Rightarrow dE^* = \delta Q$$

$$\text{Maxwell equation: } \left(\frac{\partial T}{\partial H} \right)_S = - \left(\frac{\partial m}{\partial S} \right)_H$$

Helmholtz free energy: $F = U - TS$

$$dF = dU - TdS - SdT$$

$$T = \text{cte} \Rightarrow dF = \delta W$$

$$dF = -SdT - PdV$$

$$\text{Maxwell equation: } \left(\frac{\partial S}{\partial V} \right)_T = \left(\frac{\partial P}{\partial T} \right)_V$$

$$dF = -SdT + Hdm$$

$$\text{Maxwell equation: } \left(\frac{\partial S}{\partial m} \right)_T = - \left(\frac{\partial H}{\partial T} \right)_m$$



thermodynamic relations

hydrostatic system

Gibbs free energy G and
magnetic Gibbs free energy G^*

$$G = E - TS; G^* = E^* - TS$$

$$dG = dE - TdS - SdT$$

$$dG^* = dE^* - TdS - SdT$$

$$dG = -SdT + VdP$$

$$\text{Maxwell equation: } \left(\frac{\partial S}{\partial P} \right)_T = - \left(\frac{\partial V}{\partial T} \right)_P$$

$$T = \text{cte} \text{ and } P = \text{cte} \Rightarrow dG = 0$$

$$dG^* = -SdT - mdH$$

$$\text{Maxwell equation: } \left(\frac{\partial S}{\partial H} \right)_T = \left(\frac{\partial m}{\partial T} \right)_H$$

$$T = \text{cte} \text{ and } H = \text{cte} \Rightarrow dG^* = 0$$

$$T = \text{cte} \text{ and } P = \text{cte} \Rightarrow dG = \delta W_{\text{non-mechanical}}$$

magnetic system

thermodynamic relations: TdS equation

$$S = S(T, H)$$
$$TdS = T \left(\frac{\partial S}{\partial T} \right)_H dT + T \left(\frac{\partial S}{\partial H} \right)_T dH$$
$$TdS = C_H dT + T \left(\frac{\partial m}{\partial T} \right)_H dH$$

thermodynamic relations

$$TdS = C_H dT + T \left(\frac{\partial m}{\partial T} \right)_H dH$$

adiabatic process ($dS = 0$):

$$dT_s = -\frac{T}{C_H} \left(\frac{\partial m}{\partial T} \right)_H dH$$



$$\Delta T_s = - \int_{H_i}^{H_f} \frac{T}{C_H} \left(\frac{\partial m}{\partial T} \right)_H dH$$

isothermal process ($dT = 0$):

$$dS_T = \left(\frac{\partial m}{\partial T} \right)_H dH$$



$$\Delta S_T = \int_{H_i}^{H_f} \left(\frac{\partial m}{\partial T} \right)_H dH$$

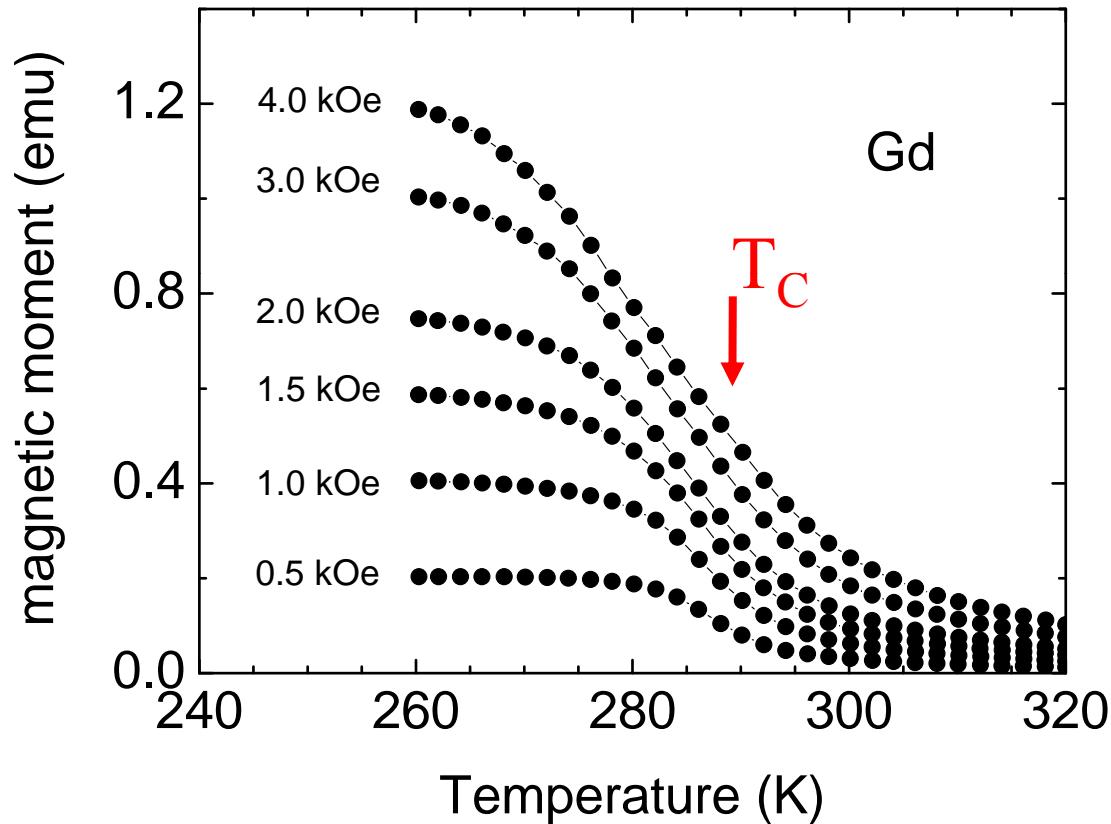
cte magnetic field ($dH = 0$):

$$dS_H = C_H \frac{dT}{T}$$



$$\Delta S_H = \int_{T_i}^{T_f} C_H(T') \frac{dT'}{T'}$$

conventional characterization: magnetic moment and entropy

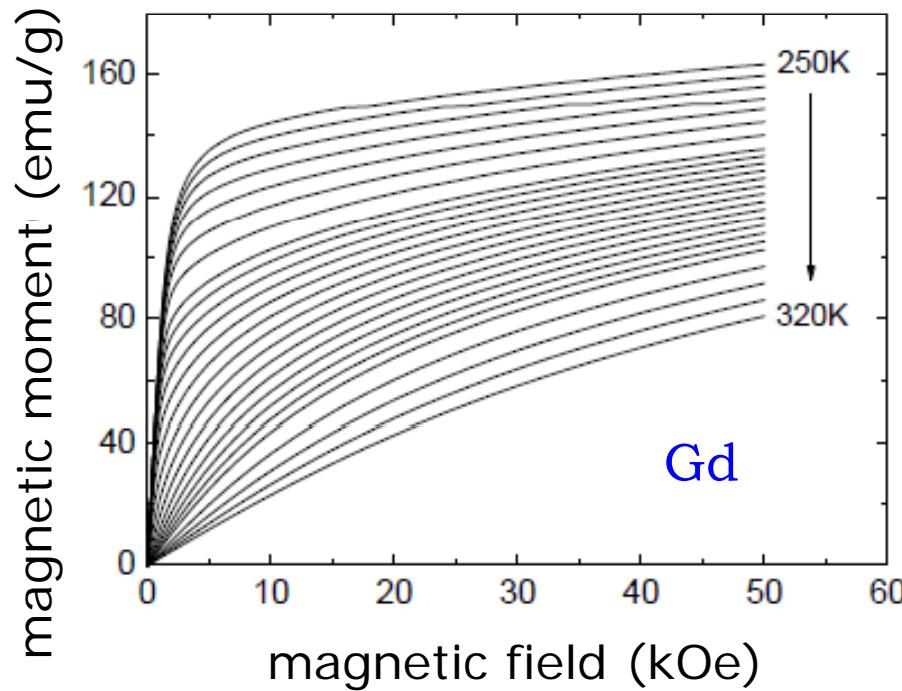


$$\Delta S_T = \int_{H_i}^{H_f} \left(\frac{\partial m}{\partial T} \right)_H dH$$

$$\Delta T_S = - \int_{H_i}^{H_f} \frac{T}{C_H} \left(\frac{\partial m}{\partial T} \right)_H dH$$

$$\Delta T_S \approx - \frac{T}{C_H} \Delta S_T$$

conventional characterization: magnetic moment and entropy

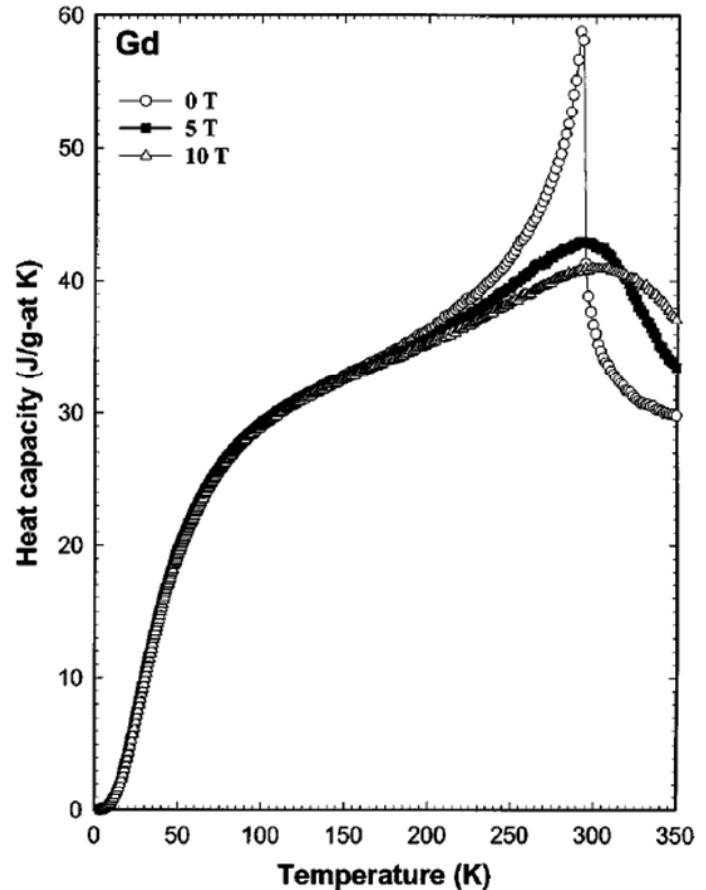
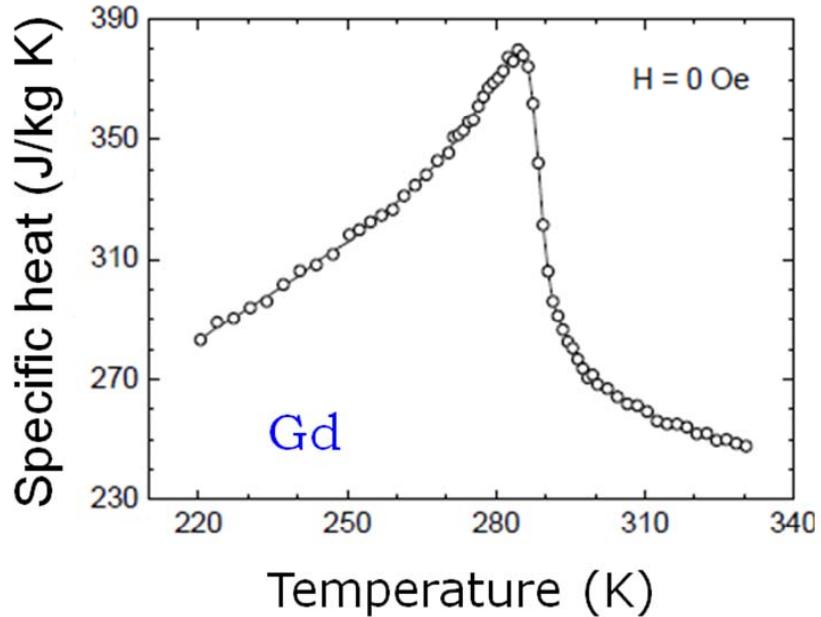


$$\Delta S_T = \int_{H_i}^{H_f} \left(\frac{\partial m}{\partial T} \right)_H dH$$

$$\Delta T_S = - \int_{H_i}^{H_f} \frac{T}{C_H} \left(\frac{\partial m}{\partial T} \right)_H dH$$

$$\Delta T_S \approx - \frac{T}{C_H} \Delta S_T$$

conventional characterization: specific heat and entropy



$$\Delta S_{T_1}^{0 \rightarrow H_2} = \int_0^T (C_{H_2}(T') - C_{H_1=0}(T')) \frac{dT'}{T'}$$

conventional characterization: other techniques

- direct measurement of the temperature (thermocouple)
- direct measurement of the entropy (heat) flow:
Seebeck (thermopower) effect

$$E_{\text{emf}} = -S \nabla T$$

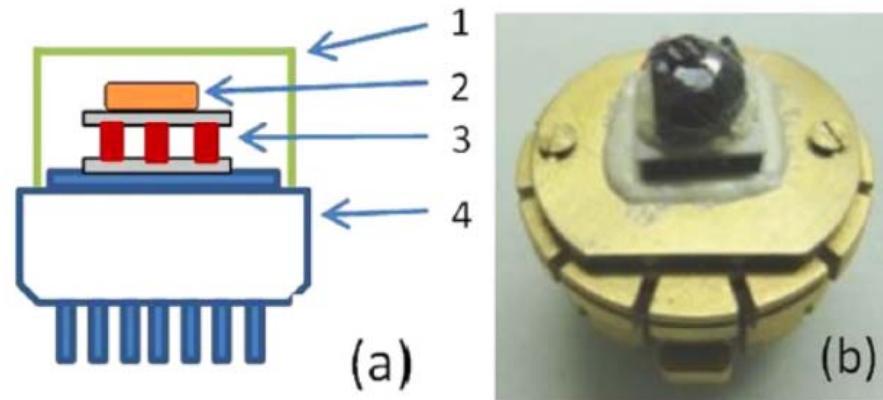


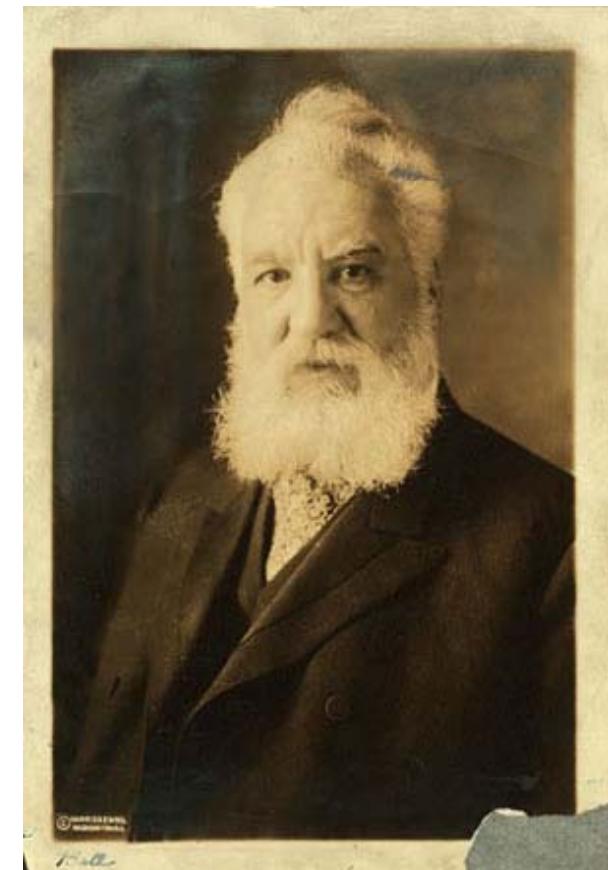
FIG. 1. (a) Schematic drawing of the Peltier element mounted on the PPMS puck: 1—heat shield; 2—sample; 3—Peltier element; and 4—puck. (b) Picture of the actual puck with a $\text{Gd}_5\text{Ge}_2\text{Si}_2$ sample.

J. C. B. Monteiro, R. D. dos Reis, A. M. Mansanares, F. C. G. Gandra,
Appl. Phys. Lett. **105**, 074104 (2014).

photoacoustic effect

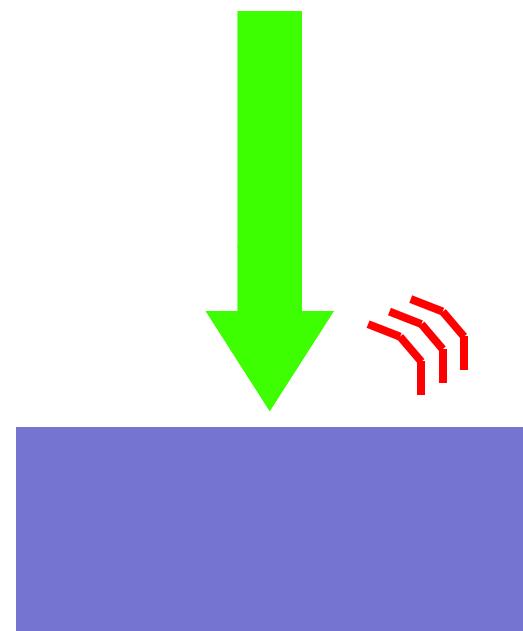


the photophone (1880):
optical fibers precursor



alexander graham bell

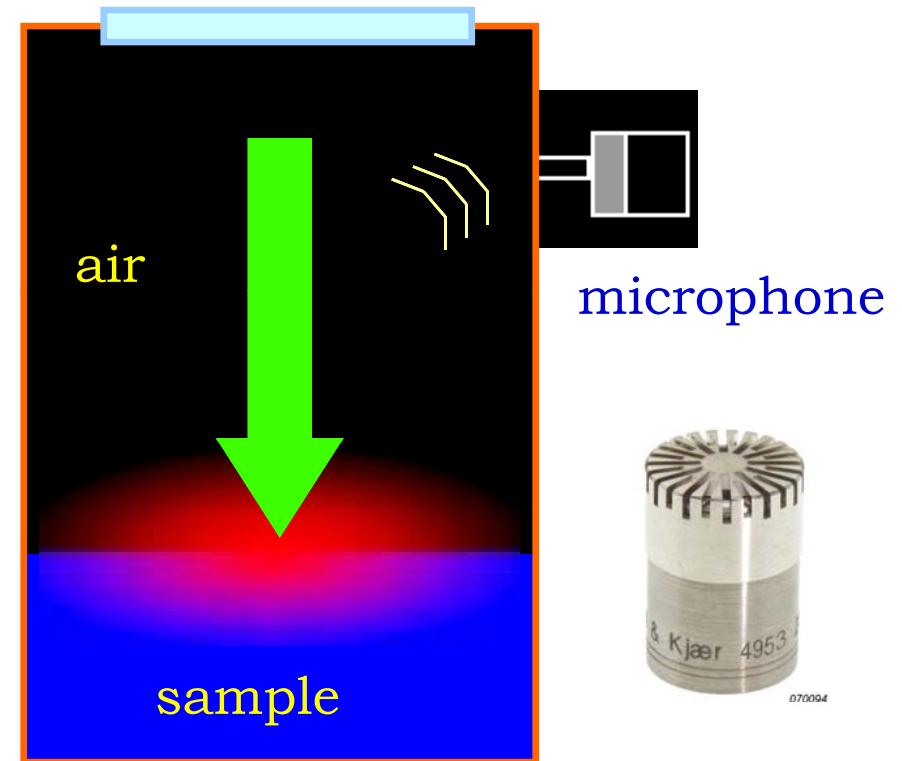
photoacoustic effect



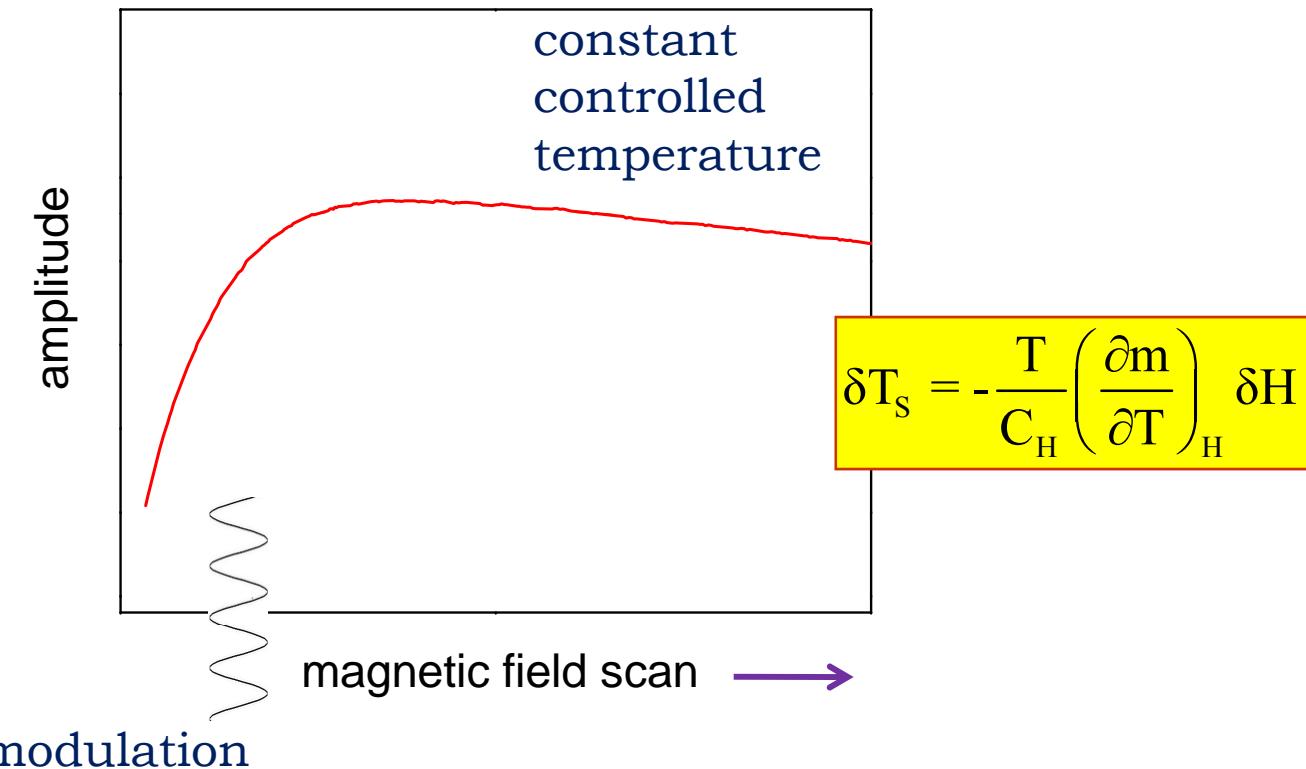
the photophone

photoacoustic effect

- light absorption
- heat propagation:
to the sample's bulk and
to the surrounding air
- surrounding air expansion
- air column compression:
acoustic wave generation

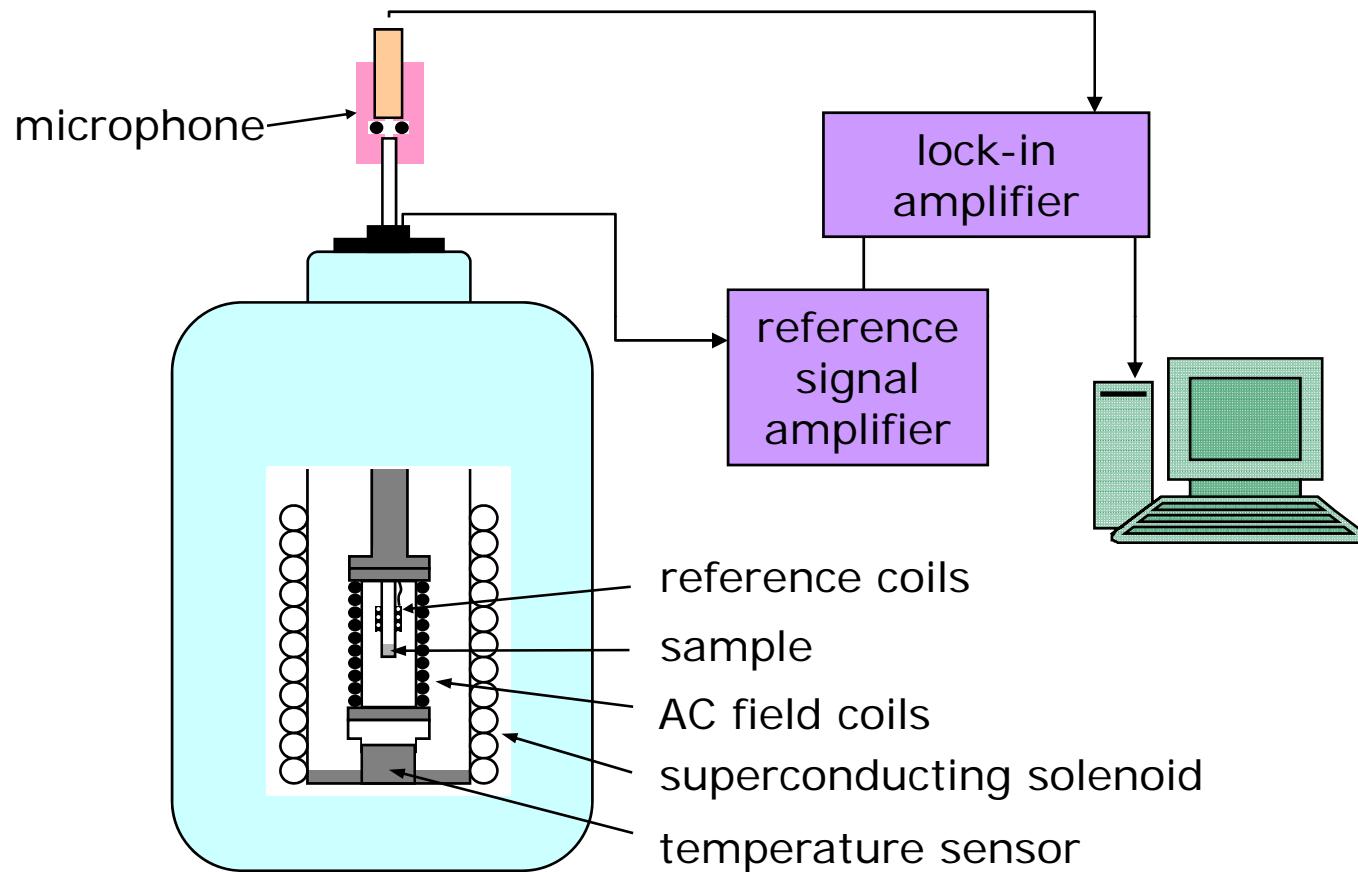


magnetoacoustic signal





experimental setup



experimental setup

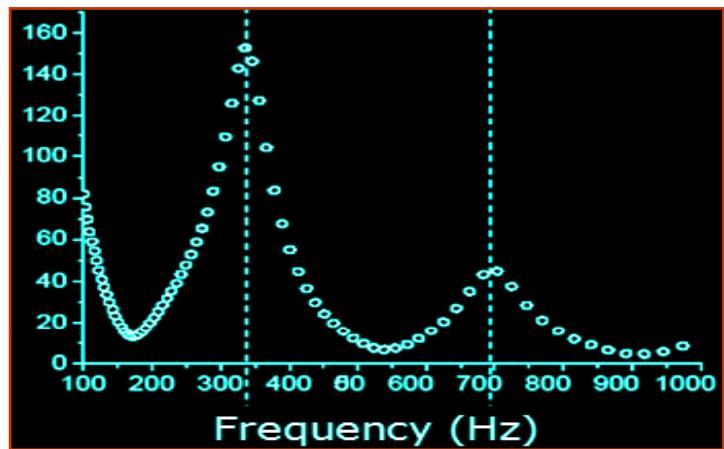


PPMS (0-50 kOe)

ESR spectrometer (0-20 kOe)

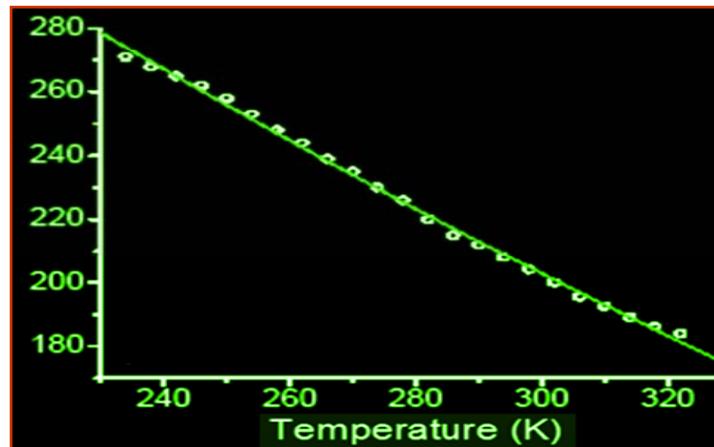


experimental conditions

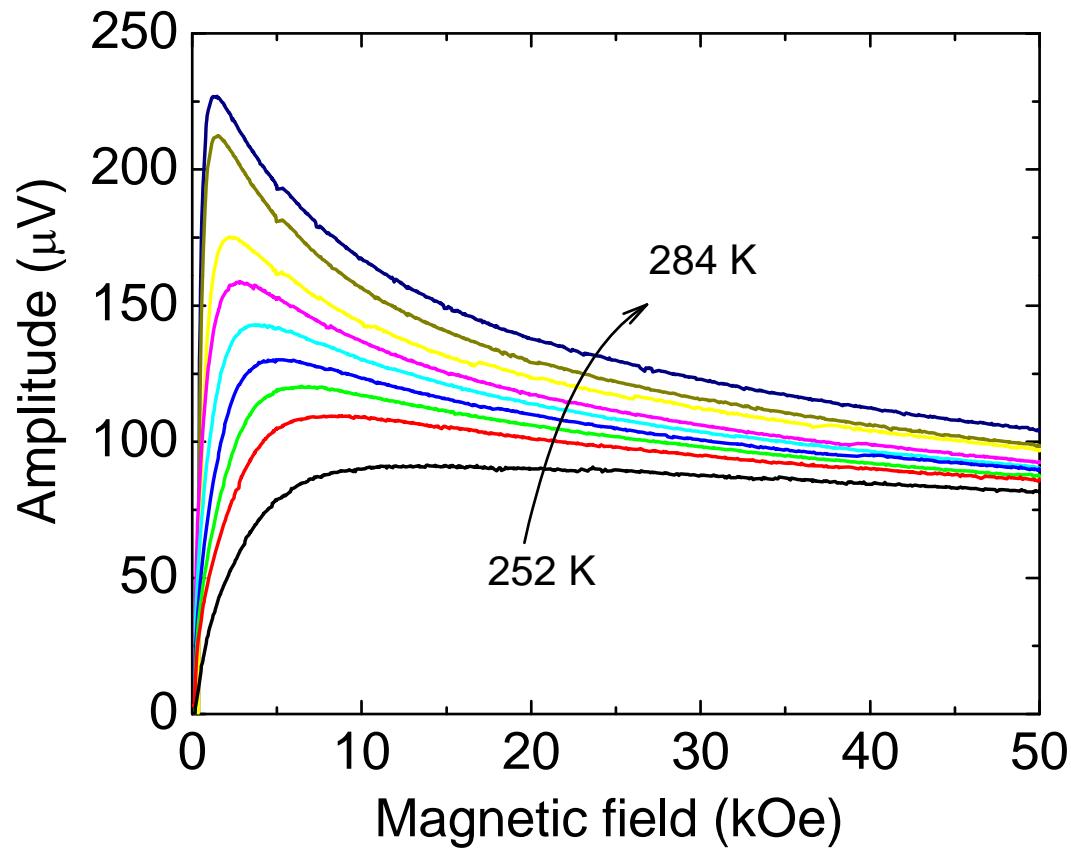


$$\delta H = 30 \text{ Oe}$$
$$f = 270 \text{ Hz}$$

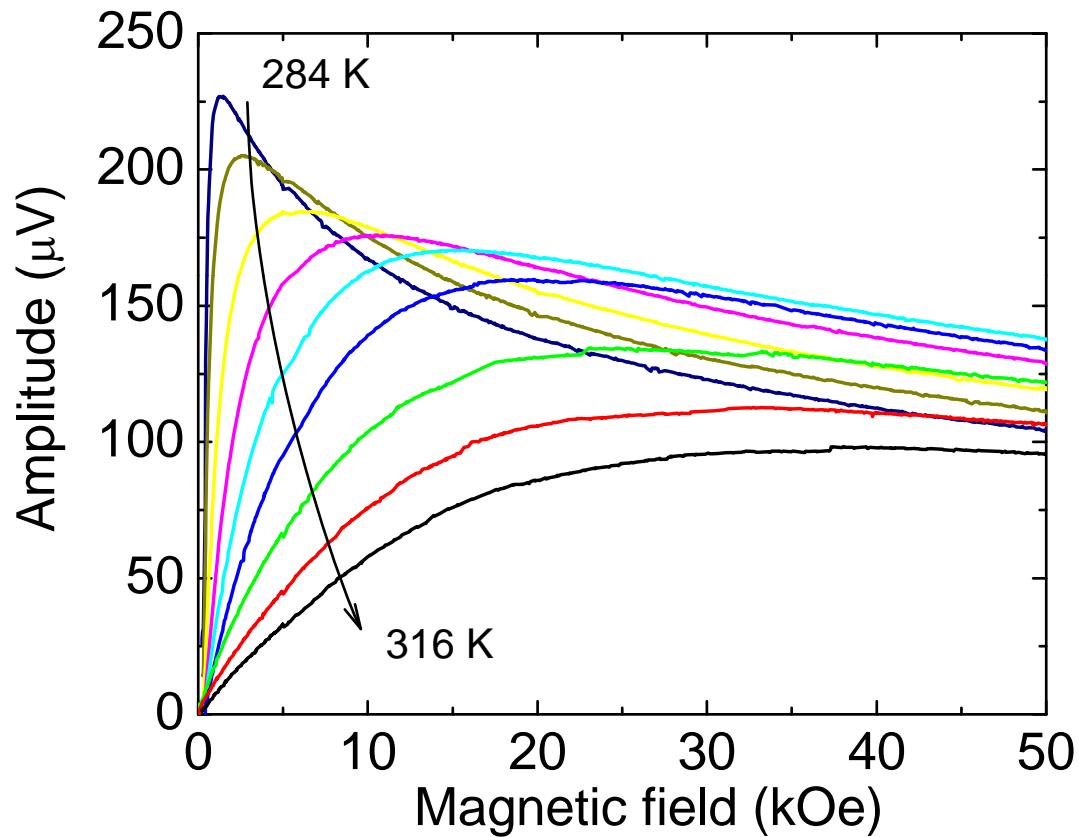
cell response



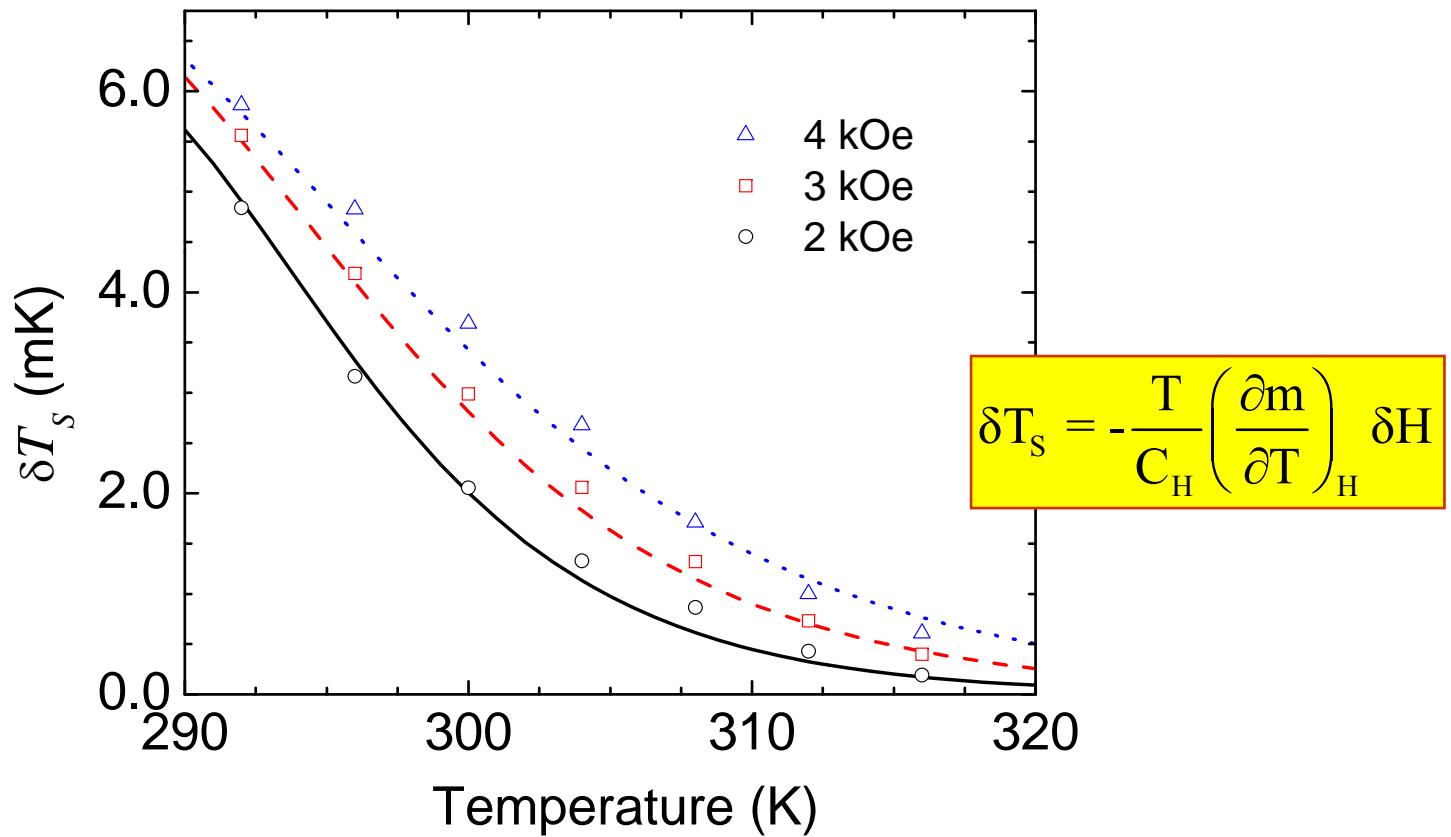
magnetoacoustic signal for Gd



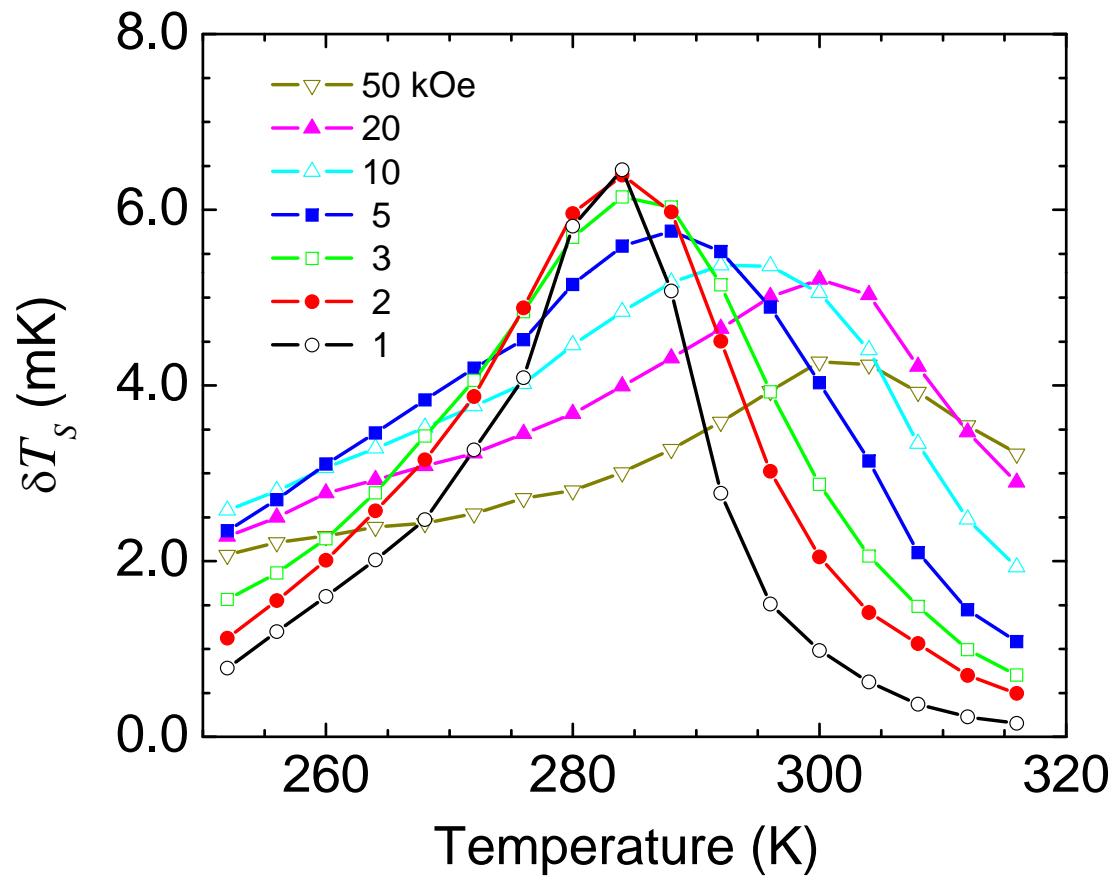
magnetoacoustic signal for Gd



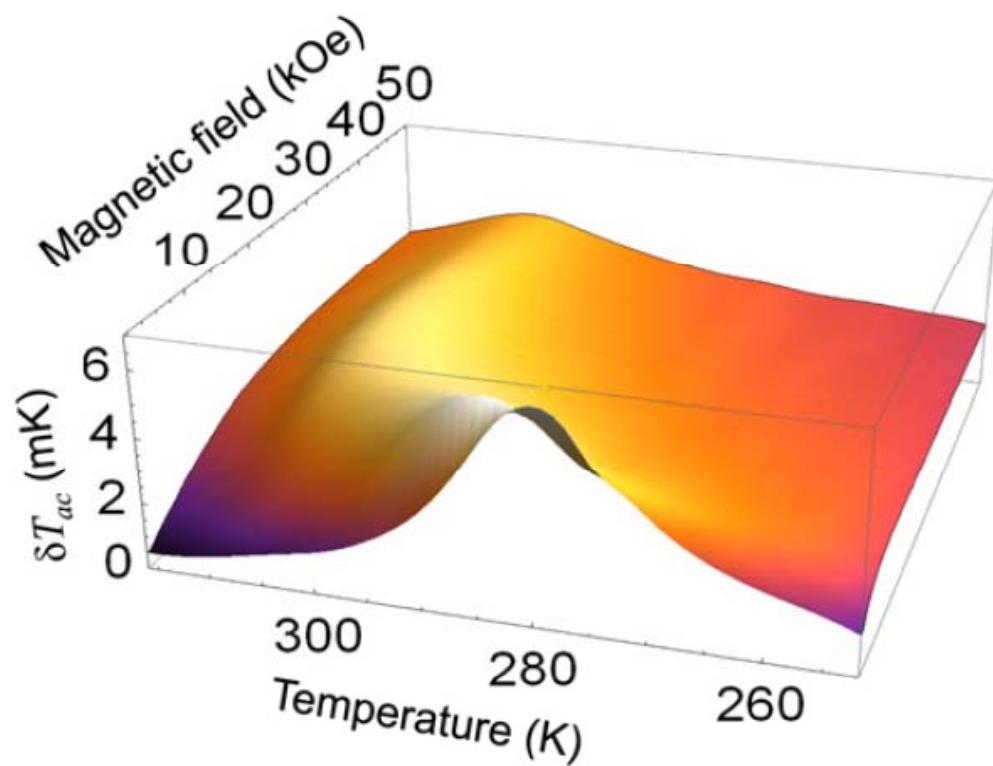
calibration using Gd reference values



temperature rise for Gd

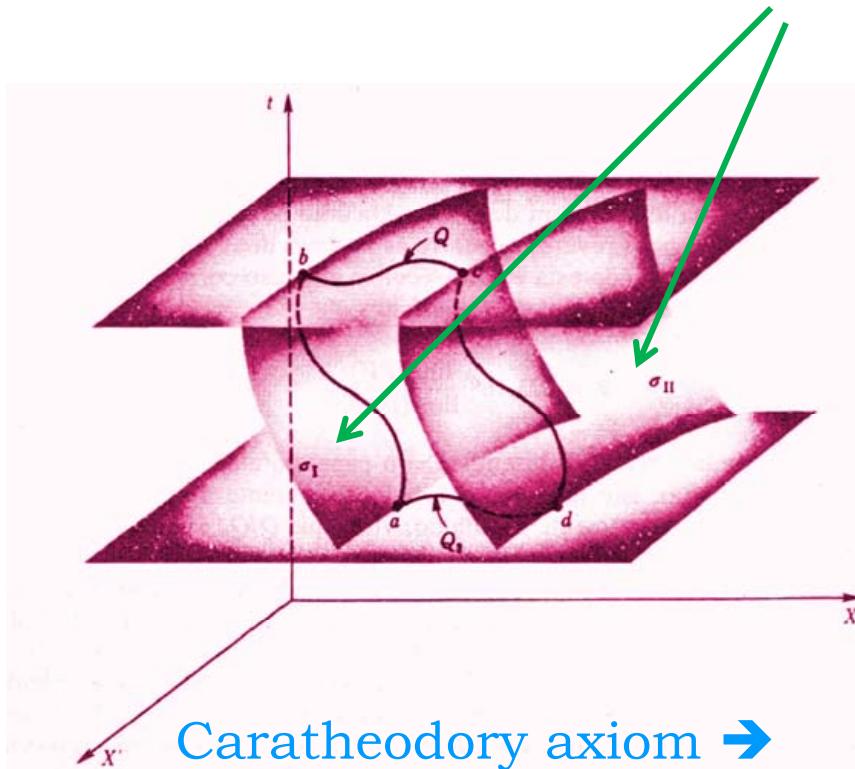


temperature map for Gd

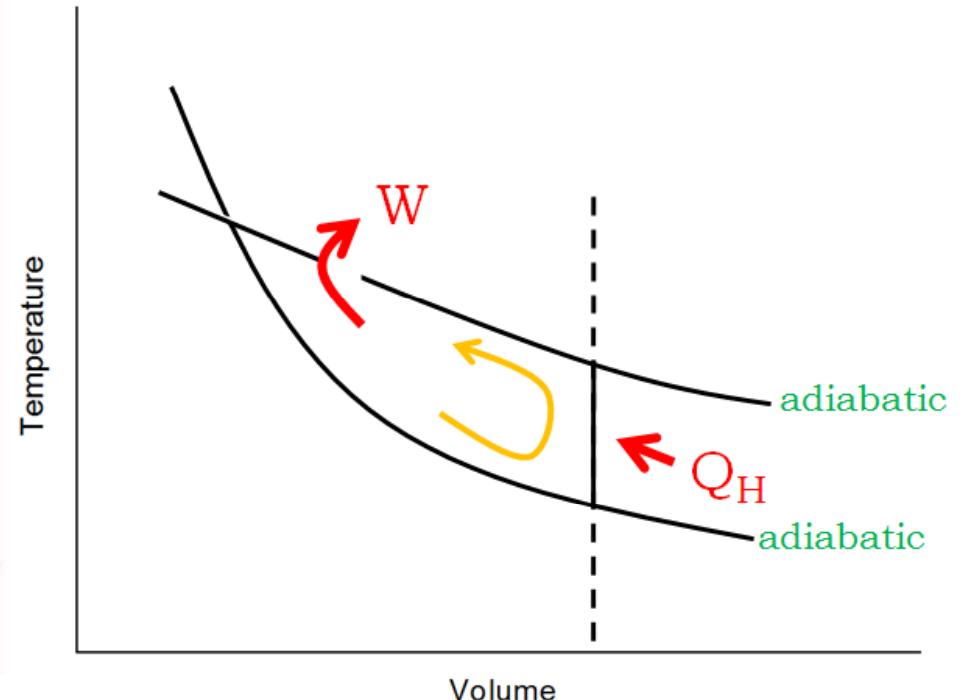


ΔT_S and ΔS_T ???

adiabatic surfaces

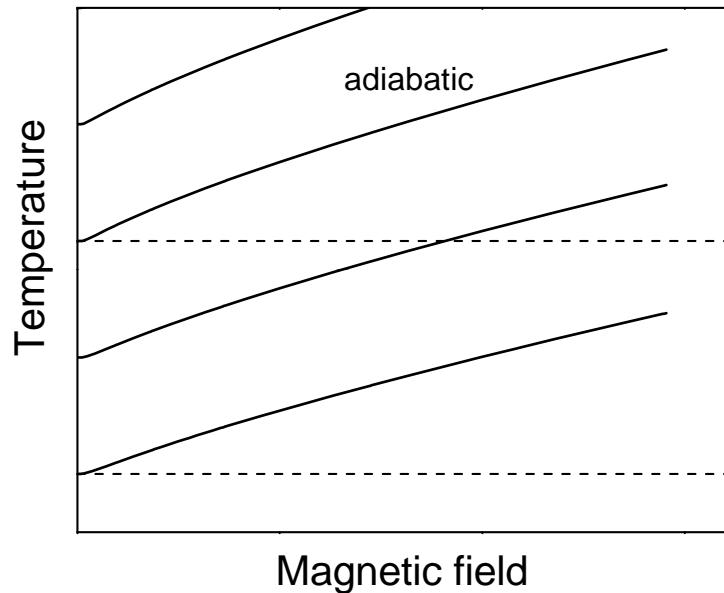


→ adiabatic surfaces
do not cross each other
(second law)



efficiency = $(W/Q_H) = 100\% \rightarrow$
→ the second law would be violated

adiabatic curves



acoustic detection of the violation of the second law of thermodynamics

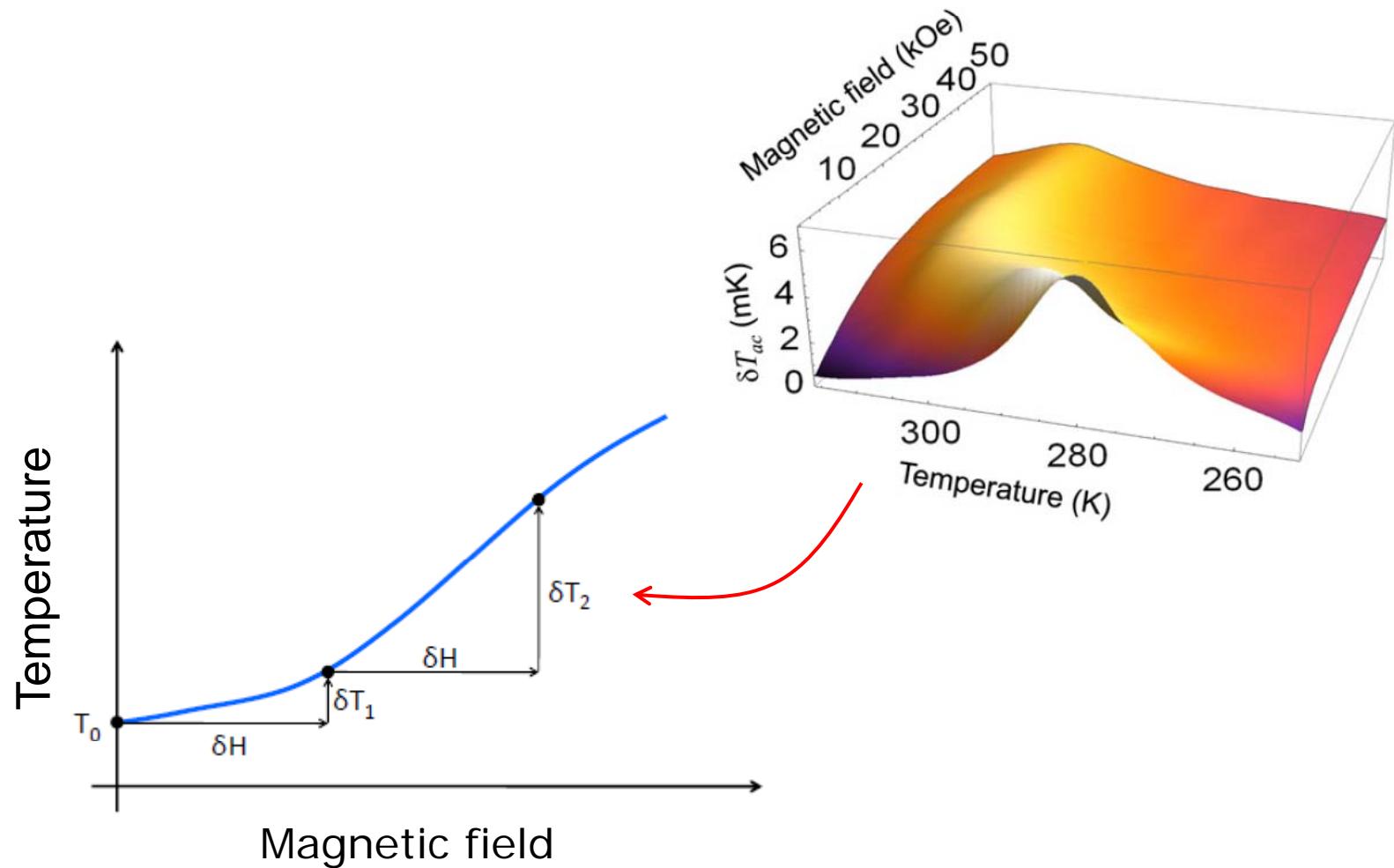
Adiabatic process:

$$dT_s = -\frac{T}{C_H} \left(\frac{\partial m}{\partial T} \right)_H dH$$

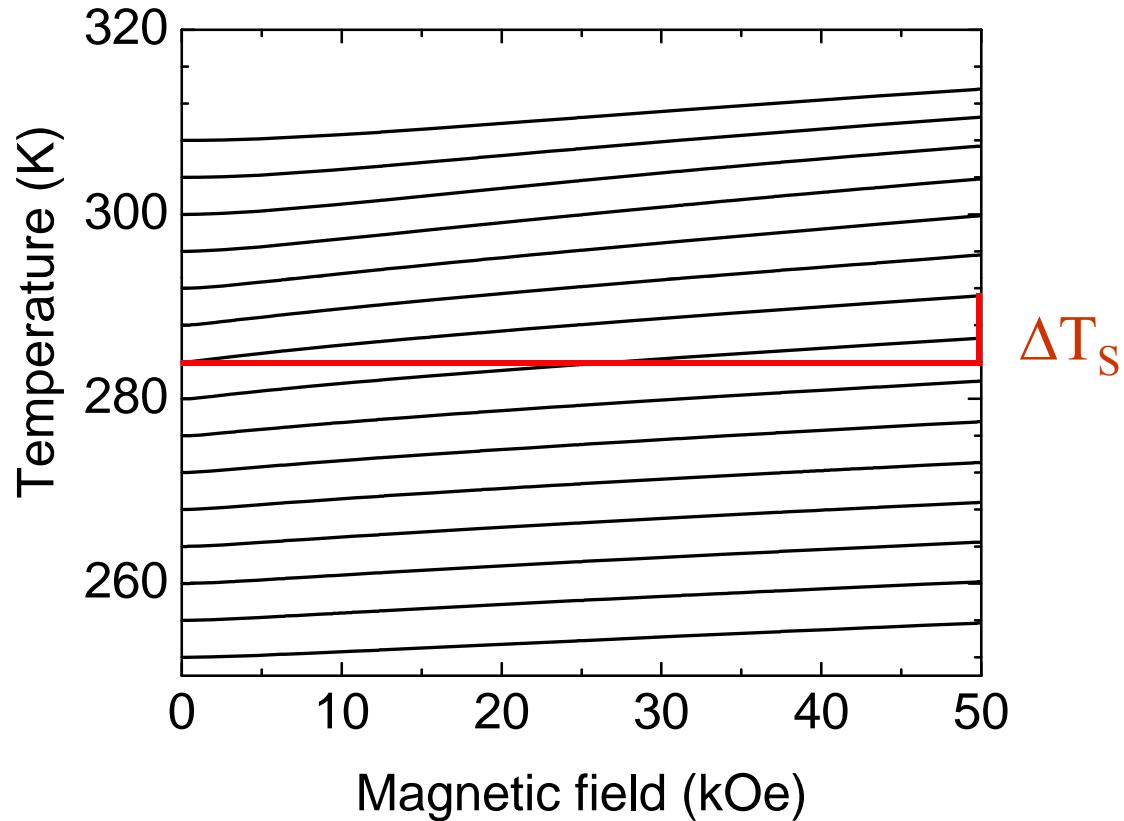
measured signal gives the slope of the adiabatic curves



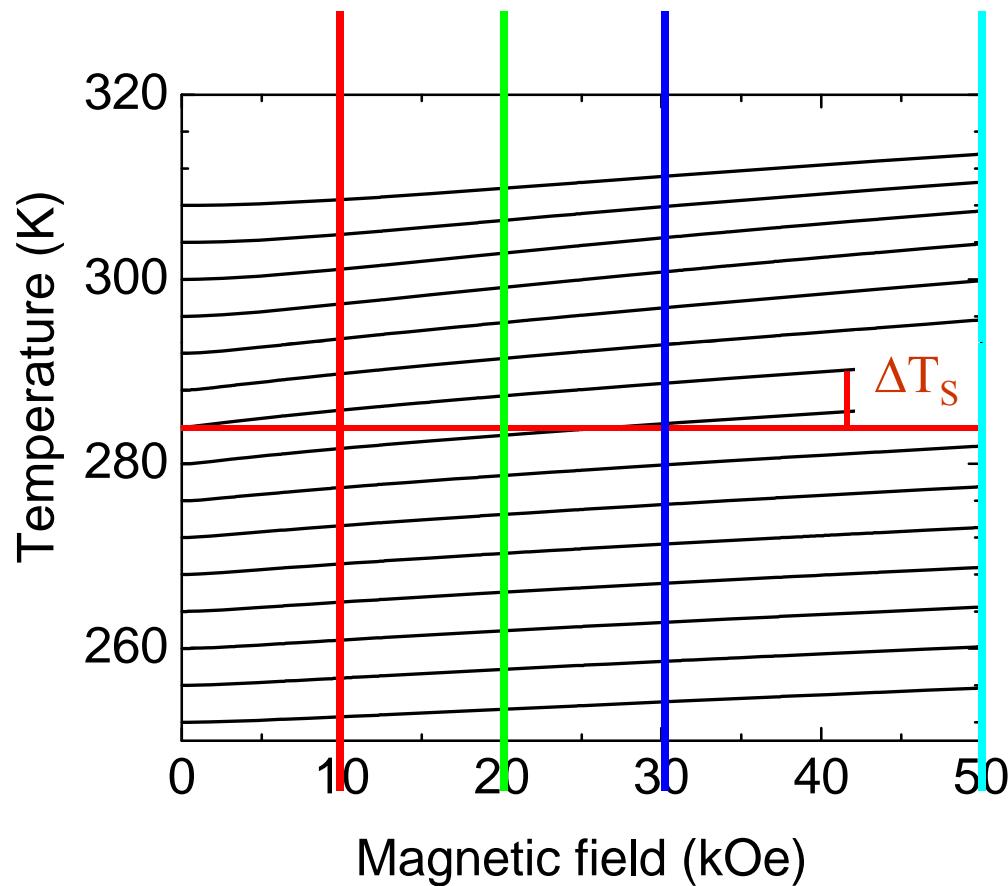
building adiabatic curves



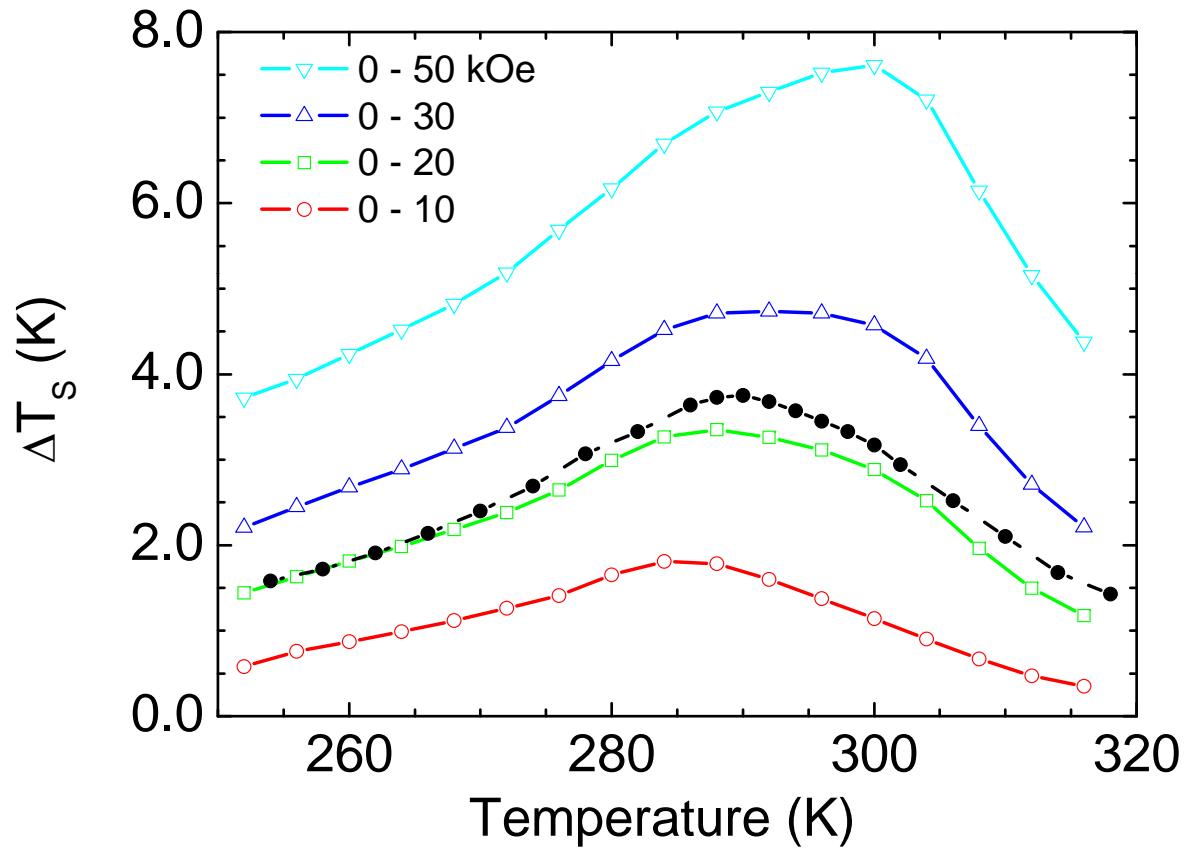
adiabatic curves for Gd

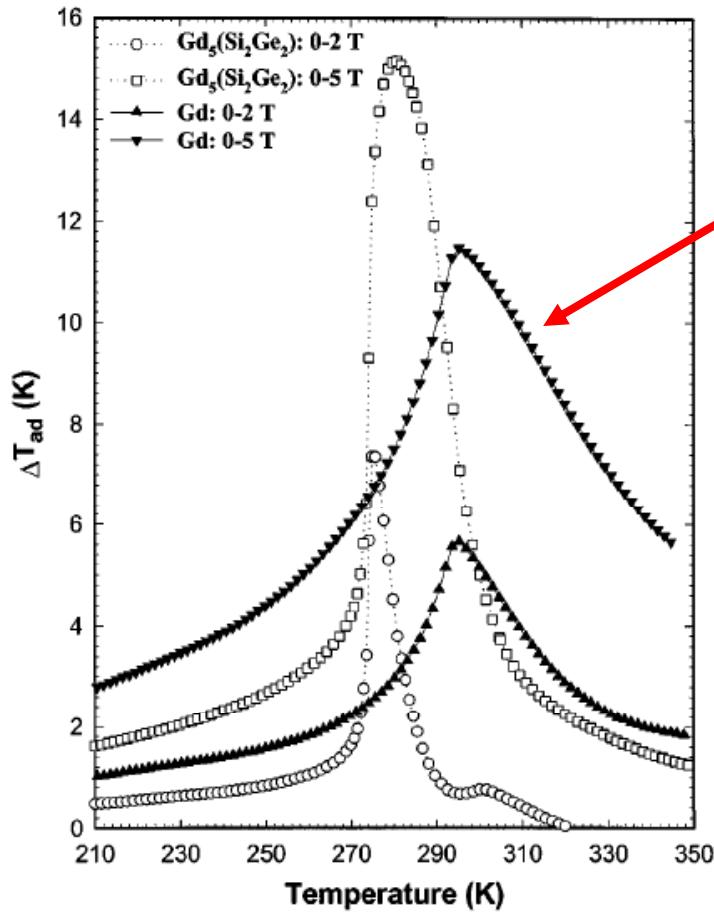


adiabatic curves for Gd



adiabatic temperature change for Gd



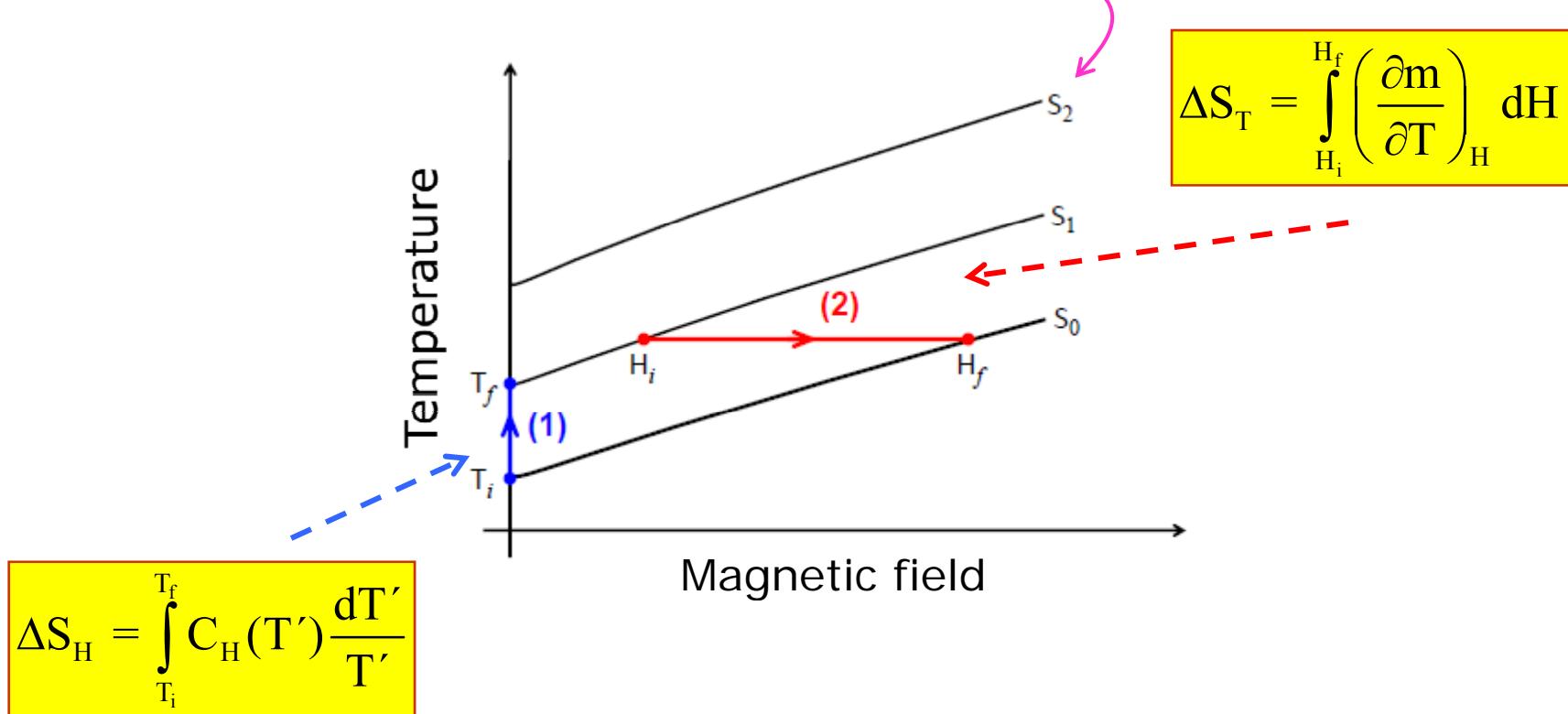


V. K. Pecharsky and
K. A. Gschneidner Jr.,
Phys. Rev. Lett. 78, 4494 (1997)

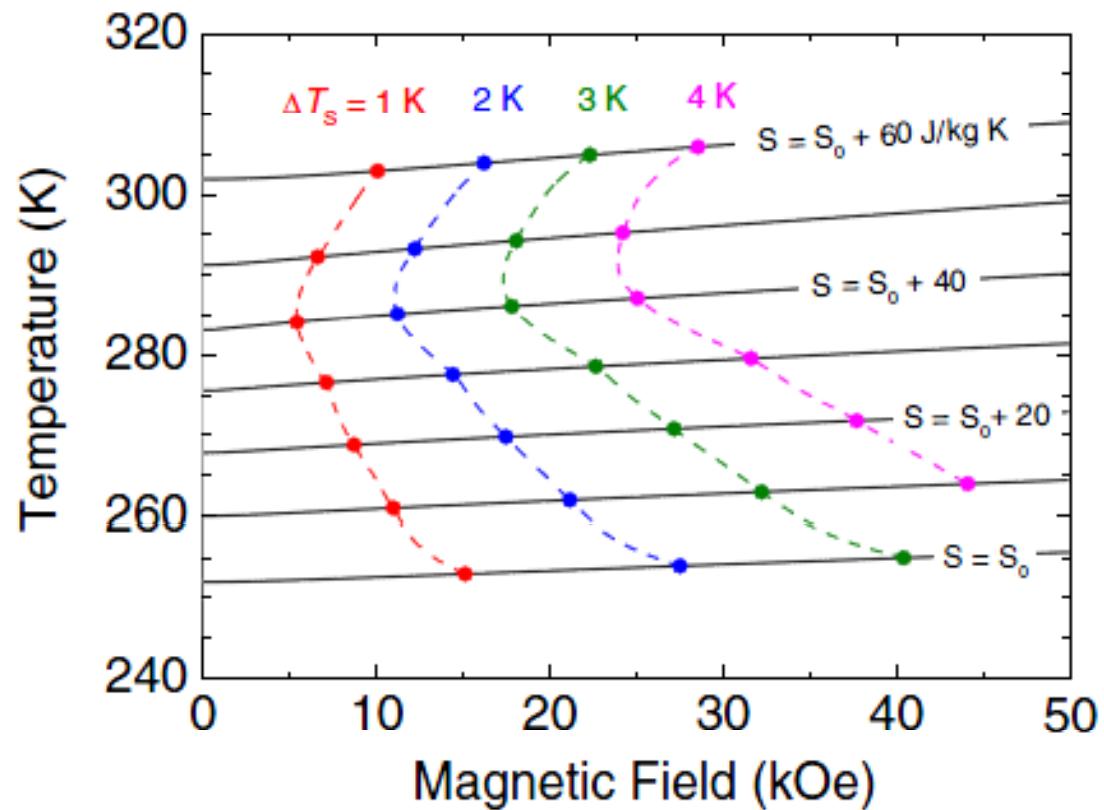
$$\Delta T_s = - \int_{H_i}^{H_f} \frac{T}{C_H} \left(\frac{\partial m}{\partial T} \right)_H dH$$

FIG. 6. The magnetocaloric effect in $\text{Gd}_5(\text{Si}_2\text{Ge}_2)$ from 210 to 350 K in comparison with that of pure Gd for magnetic field change from 0 to 2 and from 0 to 5 T.

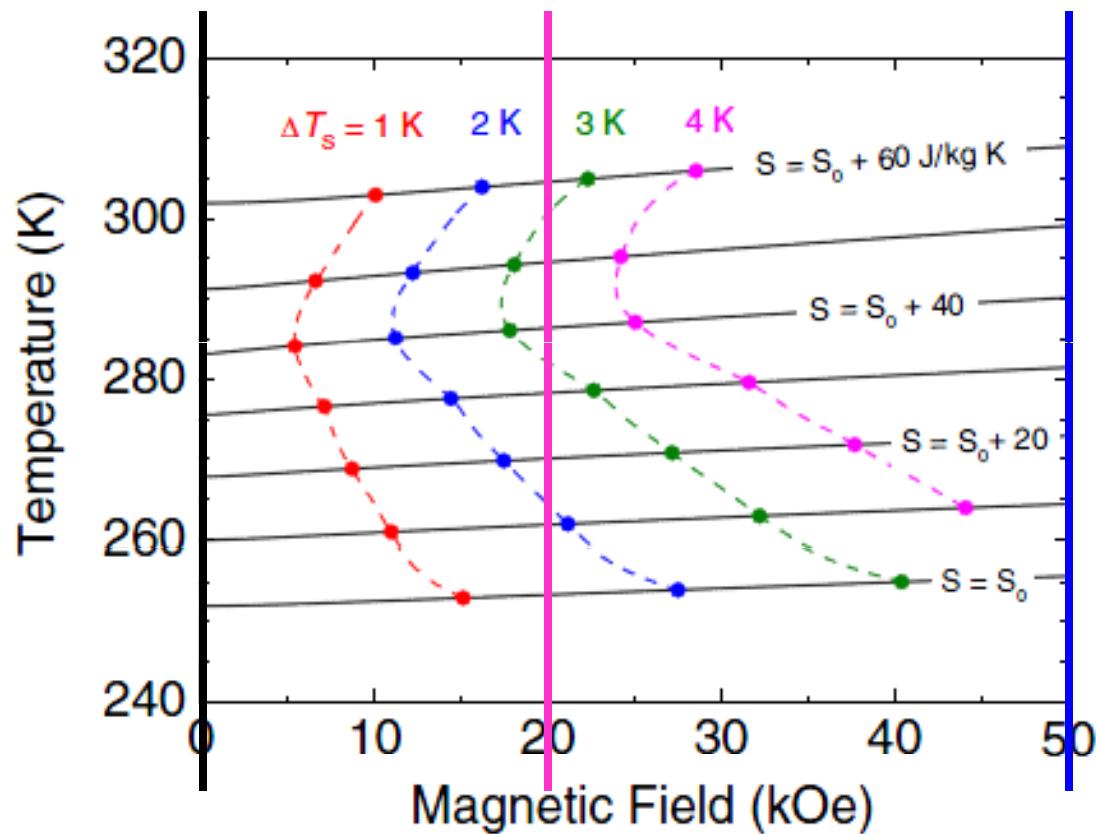
assigning the entropy



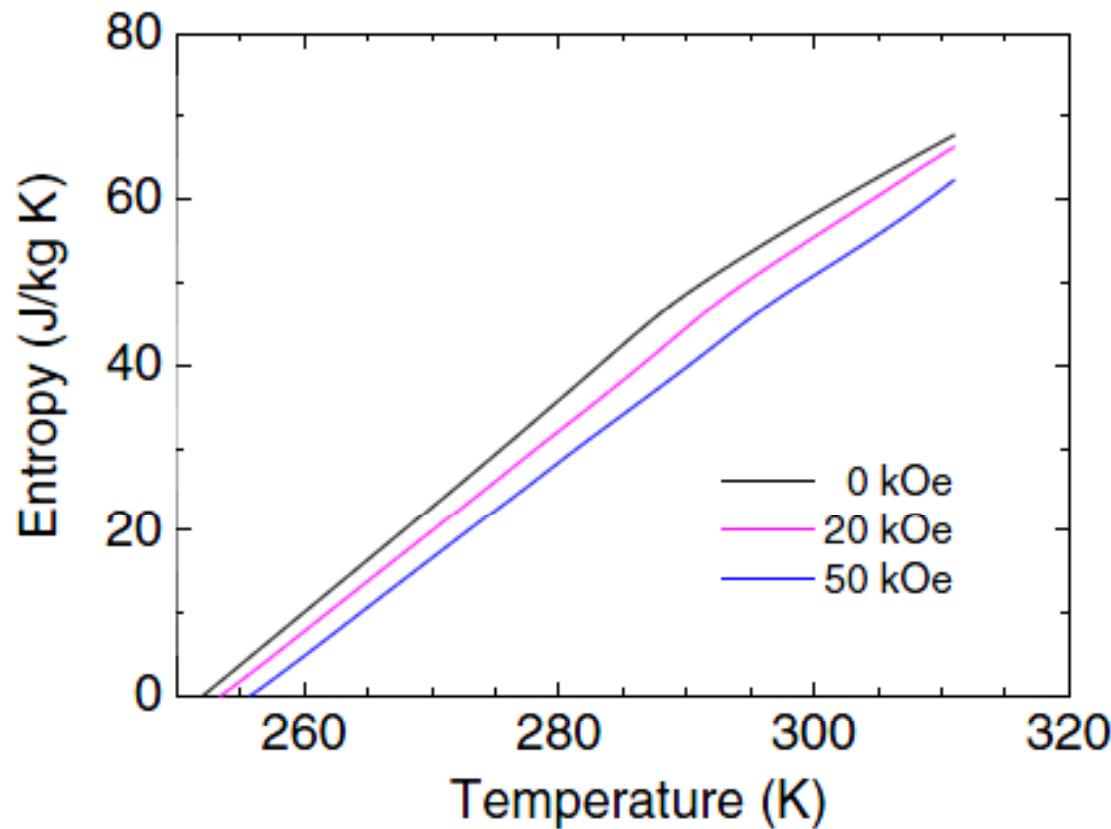
adiabatic curves for Gd



adiabatic curves for Gd

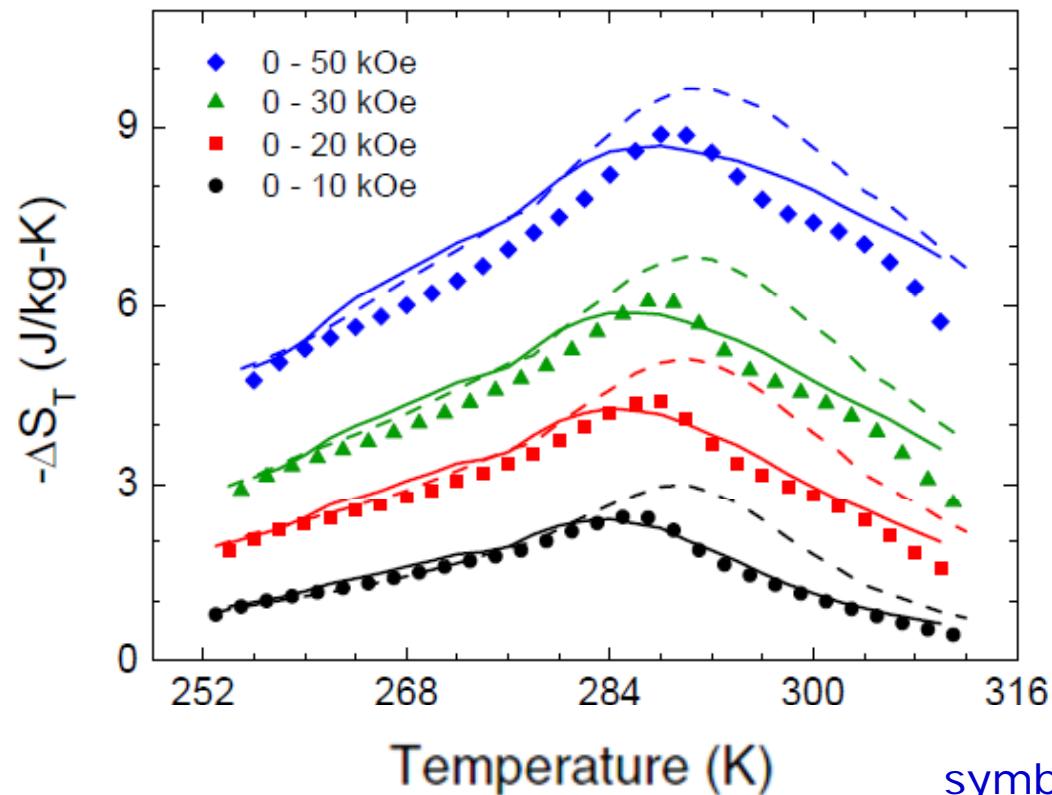


Gd entropy

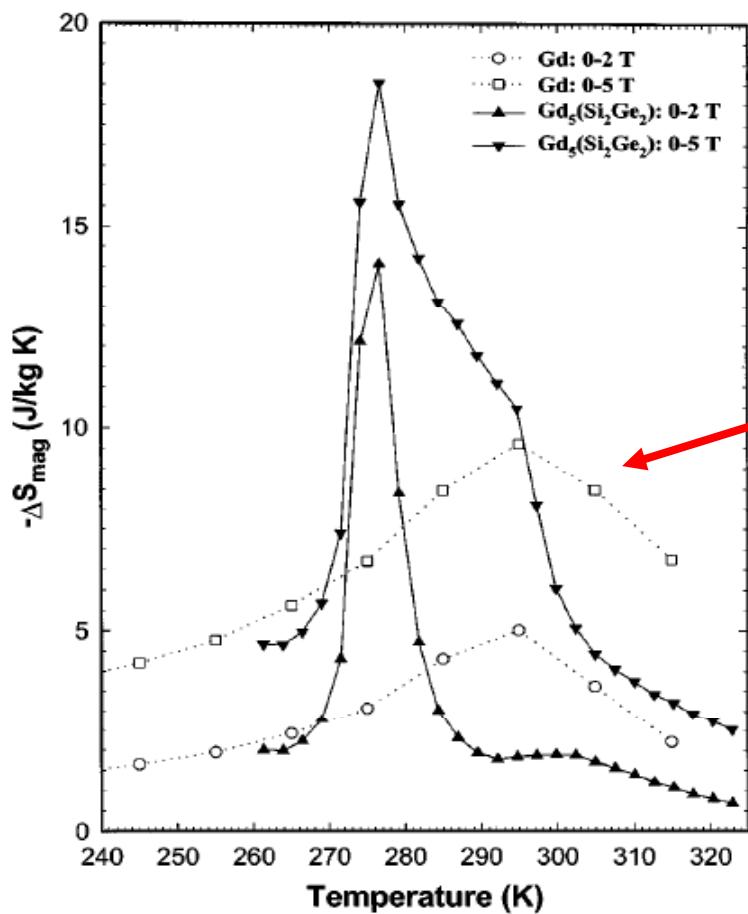




isothermal entropy change for Gd



symbols: acoustic + specific heat
solid lines: acoustic + magnetization
dashed lines: magnetization



V. K. Pecharsky and
K. A. Gschneidner Jr.,
Phys. Rev. Lett. 78, 4494 (1997)

FIG. 4. Magnetic entropy change of the $\text{Gd}_5(\text{Si}_2\text{Ge}_2)$ between 240 and 325 K for a magnetic field change 0 to 2 and 0 to 5 T, respectively, compared to that of pure Gd as determined from magnetization measurements.

acoustic detection summary

- direct determination of the adiabatic temperature variation
- calibration: no further information about the sample is necessary
- suitable for measuring the magnetocaloric effect under very small magnetic field steps (high sensitivity)
- able to measure very small amounts of samples → application to magnetic thin films
- determination of the isothermal entropy variation provided $C_H(T)$ is known for a single value of H ($H = 0$, for instance)

published articles

1. A. O. Guimarães, M. E. Soffner, A. M. Mansanares, A. A. Coelho, A. Magnus G. Carvalho, M. J. M. Pires, S. Gama, E. C. da Silva, *Acoustic detection of the magnetocaloric effect : application to Gd and Gd_{5.09}Ge_{2.03}Si_{1.88}*, Phys. Rev. B **80**, 134406 (2009).
2. A. O. Guimarães, A. M. Mansanares, A. Magnus G. Carvalho, A. A. Coelho, S. Gama, M. J. M. Pires, E. C. da Silva, *Photoacoustic based technique for measuring the magnetocaloric effect*, Journal of Physics: Conferences Series **214** (2010) 012137.
3. A. O. Guimarães, M. E. Soffner, A. M. Mansanares, A. A. Coelho, A. Magnus G. Carvalho, M. J. M. Pires, S. Gama and E. C. da Silva, *Magnetocaloric effect in GdGeSi compounds measured by the acoustic detection technique: influence of composition and sample treatment*, J. Appl. Phys. **107**, 073524 (2010).
4. M. E. Soffner, A. M. Mansanares, F. C. G. Gandra, A. A. Coelho, S. Gama, A. Magnus G. Carvalho, M. J. M. Pires, A. O. Guimarães and E. C. da Silva, *Determination of entropy change using the acoustic detection in the investigation of the magnetocaloric effect*, J. Phys. D: Appl. Phys. **43** 445002 (2010).

published articles

5. A. Magnus G. Carvalho, J. C. G. Tedesco, M. J. M. Pires, M. E. Soffner, A. O. Guimarães, A. M. Mansanares and A. A. Coelho, *Large magnetocaloric effect and refrigerant capacity near room temperature in as-cast Gd₅Ge₂Si_{2-x}Sn_x compounds*, Appl. Phys. Lett. **102**, 192410 (2013).
6. A. M. Mansanares, F. C. G. Gandra, M. E. Soffner, A. O. Guimarães, E. C. da Silva, H. Vargas, E. Marin, *Anisotropic magnetocaloric effect in gadolinium thin films: magnetization measurements and acoustic detection*, J. Appl. Phys. **114**, 163905 (2013).
7. J. C. B. Monteiro, R. D. dos Reis, A. M. Mansanares, F. C. G. Gandra, *Determination of the magnetocaloric entropy change by Field sweep using a heat flux setup*, Appl. Phys. Lett. **105**, 074104 (2014).
8. U. Nogal, A. M. Mansanares, F. C. G. Gandra, M. E. Soffner, A. O. Guimarães, E. C. da Silva, H. Vargas, E. Marín and A. Calderón, *Acoustic detection of the magnecaloric effect in gadolinium thin films: influence of the substrate*, Int. J. of Thermophysics **36**, 1099-1105 (2015).

thanks

to my students and co-workers,

to the organizing committee of the 2015 ifgw winter school,

and to the audience !!!

a. m. mansanares

2015 ifgw winter school